BSM from a Stringy Perspective

A. Hebecker (Heidelberg)

Plan: (I) SM; Hierarchy problem
(II) SUSY; GUTs; other ideas (esp. extra dims. & Kaluza-Klein th.)
(III) Strings, Fluxes, Landscape, Multiverse
(IV) String geometries & Fluxes in more detail
(V) GKP/KKLT; large-Volume scenario; intersect. branes; F-theory GUTs

General Ref.: Ibanez/Uranga: "String th. & part. phys." (book)

1) The Standard Model

(basic QFT & GR will be assumed but elem. questions are still welcome)

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \frac{1}{2}\bar{M}_P^2 R[g_{\mu\nu}] - \lambda + \mathcal{L}_{DM} + \cdots \right\}$$

↑ eff. action below, e.g., $\mu \sim 1\,\text{TeV}$

$$\bar{M}_P = M_P/\sqrt{8\pi} = 2.4 \cdot 10^{18}\,\text{GeV}$$

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermions} + \mathcal{L}_{scalars} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = -\sum_{i=1}^{3} \frac{1}{4g_i^2} \left\{ \sum_{a=1}^{d_i} F^{(i)a}_{\mu\nu} F^{(i)a,\mu\nu} \right\}$$

with $G_{SM} = U_1 \times SU_2 \times SU_3$ labelled by $i = 1,2,3$.

(We ignore $F\tilde{F}$-terms for now)

$$\mathcal{L}_{fermions} = \sum_j \bar{\psi}_j \, i \not{D} \psi_j \quad ; \quad D_\mu = \partial_\mu + i R_j(A_\mu)$$

$\bar{\psi}_j$, ψ_j: all l.h.-Dirac or Weyl spinors

R_j: G_{SM}-repres. relevant to ψ_j

$$j = \left\{ \{ q_L^a, u_R^a, d_R^a, l_L^a, e_R^a \}, \; a = 1,2,3 \right\}$$

a: families

To simplify notation: $\psi_{q_L^a} \longrightarrow q_L^a$

Crucial exp. fact:

Weyl-fermion	SU_3	SU_2	$U_1 = U_{1,Y}$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	1/6
u_R	$\bar{3}$	1	−2/3
d_R	$\bar{3}$	1	1/3
$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	2	−1/2
e_R	1	1	1

"singlet" "charge 1"
(singlet would mean "0")

$$\mathcal{L}_{\text{Scalar}} = -(D_\mu\phi)^+(D^\mu\phi) - V(\phi)$$

$$V(\phi) = -\mu^2\phi^+\phi + \lambda(\phi^+\phi)^2 \quad ; \quad \underbrace{\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}}$$

with quantum numbers $(1, 2, -1/2)$, exactly like lepton doublet.

$\mathcal{L}_{\text{Yukawa}}$ = all gauge & Poinc. inv. terms of type $\bar{\psi}\psi\phi$ & $\bar{\psi}\psi\phi^*$

$$= Y_u^{ab} q^a u^b \phi^* + Y_d^{ab} q^a d^b \phi + Y_e^{ab} \ell^a e^b \phi + \text{h.c.}$$

[This is Weyl notation; in Dirac notation one has $\overline{(q^a)_c}\, u^b$ etc.]

Problem: Check that $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$ with $v = \mu/\sqrt{2\lambda}$ extremizes pot. & leaves $\underline{a}$ U_1 ($\equiv U_{1,\text{em}}$) unbroken. Check that $Q = Y + T_3$ for this U_1 & calculate the charges of all particles. Get W & Z masses etc. etc. ...

- Think in terms of mass-dims. of objects:

$$[S] = 0 \,;\; \left[\int d^4x\right] = -4 \,;\; [\mathcal{L}] = 4 \,;\; [\partial_\mu] = 1$$
$$[\phi] = 1 \,;\; [\psi] = 3/2 \,;\; [Y^{ab}] = 0 \,;\; \ldots$$

- <u>Crucial</u>: μ^2 with $[\mu^2] = 2$ is the <u>only</u> dim.ful param.

- No renormalizable terms ($\equiv$ operators with $[\cdot\cdot] \leq 4$) can be added to the above Lagr.

- The <u>only</u> dim.-5-operator which can be added is

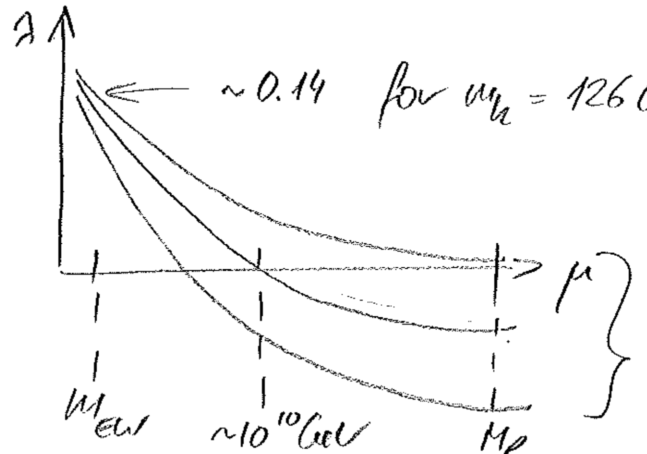
$$\mathcal{L}_W = \frac{1}{M}(\ell\phi^*)^2, \qquad (\text{"Weinberg operator"})$$

which for $M \sim 10^{15}$ GeV gives ν-masses of the right order.
$\Rightarrow M$ is highest scale up to which $\mathcal{L}_{SM}$ can be valid.
[Validity range can be extended by adding ν_R with mass at (or below) this "seesaw" scale of $10^{10} .. 10^{15}$ GeV.
Integrating out $\nu_R \rightarrow \mathcal{L}_W$]

<u>Refs.</u>: Cheng/Li ; Donoghue/Golowich/Holstein ; Ramond ; Langacker ; "Weinberg II"

- $\mathcal{L}_{SM} + \mathcal{L}[\nu_R]$ could, in principle, be valid up to M_P

<u>Caveat</u>:



~ 0.14 for $m_h = 126$ GeV $\rightarrow$ implies $\mu^2 \simeq (100\text{ GeV})^2$ for $v \simeq 170$ GeV

error range due to mostly m_t (& also α_s, m_h)

"high-scale SUSY?" (related to Strings?)

"vac. instability"

- So why look beyond the SM?
  - A) Gravity (not renormalizable)
  - B) DM
  - C)

2) Hierarchy problem: (... at least 3 faces)

① "large hier. problem"

Why μ ($\simeq m_{EW}$) $\ll M_P$

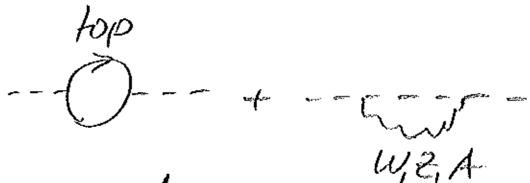
(naively, one & only fund. scale $\equiv$ scale of quant. gravity)

... but this may be just aesthetical ...

② Fine tuning

Assume QFT $\subset$ {finite theory, e.g. strings, at scale Λ},
i.e. physical cutoff Λ

$\Rightarrow \mu^2 = \mu_0^2 + \delta\mu^2$; $\delta\mu^2 \sim$



$$\sim 3 y_t^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \, \mathrm{tr}\left(\frac{1}{\not{k}+m_t}\right)^2$$

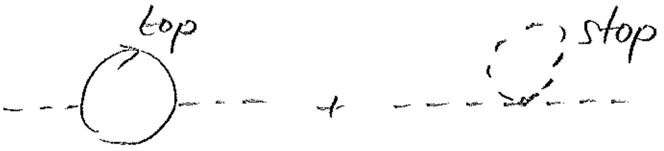
$$\sim \frac{3y_t^2}{16\pi^2} \int^{\Lambda} |k| \, d|k| \sim \frac{3y_t^2}{16\pi^2} \Lambda^2$$

If $\Lambda \sim M_P$, enormous cancellation between μ_0^2 & $\delta\mu^2$ needed!

③ Little hier. problem

Let SM be replaced by "nicer" theory in which μ^2 is not UV-divergent (e.g. SUSY) at scale $\ll M_P$

• more technically: (using SUSY example)



$= 0$ if SUSY unbroken

But, with $m_t = 0$ & $m_{\tilde{t}} \neq 0$:

need more care, incl. supersymm. regular., to see that no log. div. in mass^2 is left

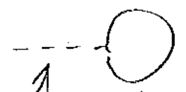
$$\int dk^2 - \int \frac{dk^2 (k^2)}{k^2 + m_{\tilde{t}}^2} \sim m_{\tilde{t}}^2 \equiv \Lambda^2$$

$\Rightarrow$ to have no fine-tuning, need $\delta\mu^2 \sim \mu^2 \sim (100\ \text{GeV})^2$

$\Rightarrow$ need $\Lambda \sim \frac{4\pi}{\sqrt{3}}\mu \sim 700\ \text{GeV}$

But LEP, FCNCs and now (partially) LHC don't like new particles at such low energies.

<u>Note:</u> ① & ② also apply to <u>cosm. const.</u>

$\lambda \sim$  $\sim \int d^4k$

$g_{\mu\nu}$ — SM particles

(a possible way out and hence a problem like ③ do apparently not even exist!)

3) SUSY

• New symm.: bosons $\leftrightarrow$ fermions

• Derivable from new symm.: $x^\mu \longleftrightarrow \underbrace{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}}_{\text{Weyl spinors \& Grassmann var.}}$

coordinates on superspace

fields $\phi(x) \longrightarrow$ superfields $\phi(x, \theta, \bar{\theta})$

- Under certain natural restrictions, one has so-called "chiral" SFs depending only on θ (not $\bar\theta$)

$$\underbrace{}\quad \phi = \phi(x,\theta)$$

→ Wess / Bagger
Terning
S. Martin (~ SPIRES)

- Taylor-exp.: $\phi(x,\theta) = A(x) + \sqrt{2}\,\theta^\alpha \psi_\alpha(x) + \theta^\alpha\theta_\alpha F(x)$

(nothing else since $\theta = (\theta^1, \theta^2)$ & $(\theta^1)^2 = (\theta^2)^2 = 0$)

- Simplest interacting model (Wess-Zumino):

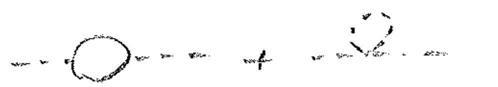
$$\mathcal{L} = \int d^2\theta\, d^2\bar\theta\, \underbrace{K(\phi,\bar\phi)}_{\text{(real) Kähler potential}} + \int d^2\theta\, \underbrace{W(\phi)}_{\text{(holomorphic) superpotential}} + \text{h.c.}$$

$$= \int d^2\theta\, d^2\bar\theta\, \phi\bar\phi + \int d^2\theta\, \frac{\lambda}{3}\phi^3 + \text{h.c.}$$

$$= i\partial_\mu\bar\psi\,\bar\sigma^\mu\psi + |\partial A|^2 + |F|^2 + \lambda\left(A^2F + \psi^2 A\right) + \text{h.c.}$$

$$= \quad \cdots \quad + \quad \cdots \quad + \underbrace{\lambda\psi^2 A - \lambda^2|A|^4}$$

$\Downarrow$



$= 0$

- Easily generalizes to gauge fields (A_μ, λ) & to full SM (→ "MSSM"): $q, u, d \longrightarrow \underbrace{Q, U, D, \cdots}_{\text{SFs}}$

$$\mathcal{L}_{\text{MSSM}} \supset \int d^2\theta\, d^2\bar\theta\, \left(Q\bar Q + U\bar U + H_u \bar H_u + \cdots\right)$$

$$+ \int d^2\theta\; Y\, Q U H_u + \cdots$$

- $m_{\tilde{t}} \gg m_t$ due to extra SUSY-terms
  (can be read as "spont. symm. breaking")
- but: little hier. probl. remains hard - much ongoing work!

4) GUTs

- Recall SU_N gauge th.: $A_\mu \in \underbrace{\mathrm{Lie}(SU_N)}_{\text{hermitian, traceless } N\times N \text{ matrices}}$

[Recall that
$U = e^{iT} \in SU_N$
iff $T \in \mathrm{Lie}(SU_N)$]

- $SU_5 \supset SU_3 \times SU_2 \times U_1$
  $\mathrm{Lie}(SU_5) \supset \mathrm{Lie}(SU_3) \oplus \mathrm{Lie}(SU_2) \oplus \mathrm{Lie}(U_1)$

$$\left(\begin{array}{c|c} 3\times 3 & \\ \hline & 2\times 2 \end{array}\right) \Longrightarrow SU_5 \supset SU_3 \times SU_2$$

- The U_1-generator is $T_{U_1} \sim \left(\begin{array}{ccc|cc} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ \hline & & & 3 & \\ & & & & 3 \end{array}\right)$

(traceless & commutes with SU_2, SU_3; in fact, unique!)

- Crucial concept: "Branching rule" of reprs

$$SU_5 \supset SU_3 \times SU_2 \times U_1$$

$$5 = \underset{\uparrow}{(3,1)_{-2}} + \underset{\uparrow}{(1,2)_3} \quad \Big\}$$ This is a common and (hopefully) self-explanatory notation

here we only care about relative normalization

"proof": $\left(\begin{array}{c|c} 3\times 3 & \\ \hline & 2\times 2 \end{array}\right) \times \left(\begin{array}{c} 3 \\ \hline 2 \end{array}\right) \Big\} 5$

- Analogously: $\bar{5} = (\bar{3},1)_2 + (1,2)_{-3}$

$$10 = (3,2)_1 + (\bar{3},1)_{-4} + (1,1)_6$$

antisymm. tensor of SU_5, i.e.

$$X^{ij} \to U^i{}_k U^j{}_\ell X^{k\ell} \; ; \; U \in SU_5 \; ; \; X^{ij} = -X^{ji}$$

problem: Work this out!

- Up to proper U_1-normalization (work this out!), it is now apparent that $(10 + \bar{5}) =$ (1 SM generation of Weyl fermions or chir. SFs)

- <u>GUT idea</u>:

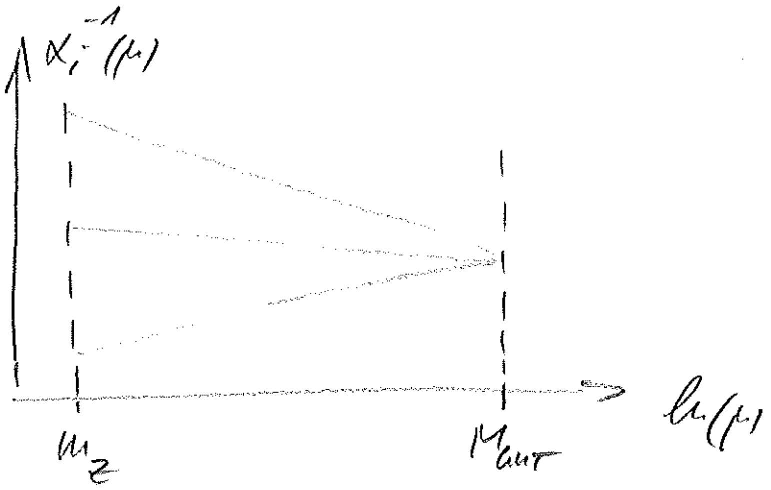
$$SU_5 \xrightarrow[\sim 10^{16}\,\text{GeV} \\ \text{(in SUSY)}]{\langle H \rangle \neq 0} SU_3 \times SU_2 \times U_1 \xrightarrow[10^2\,\text{GeV}]{\langle \phi \rangle \neq 0} SU_3 \times U_{1,EM}$$

- <u>In</u> TeV-scale SUSY:

$\Downarrow$

extra exp. support for SUSY through "α_s-prediction"

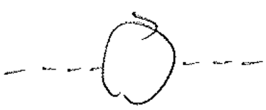
(In SM, extra structure needed to properly unify)



5) <u>Other ideas for solving the hier. problem</u>

<u>5.1 Technicolor</u>

The Higgs is a bound state of a strongly-coupled ("technicolor") gauge th. Hence the divergence in

 is automatically cut off at techn. scale.

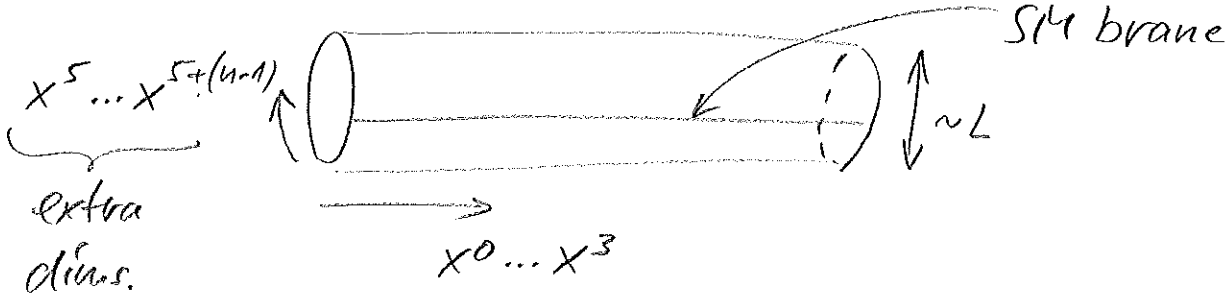
5.2 The Higgs is a pseudo-Goldstone-Boson

("Little Higgs"); recall:

[Flatness of pot. is violated at least by SM gauge interactions & top-Yuk.; extra states need to come in to prevent UV-div.; "little-hier.-probl." as in SUSY.]

5.3 large extra dims. ("ADD")

We live in a Kaluza-Klein-th. ($M^d = \mathbb{R}^4 \times C^n$) with SM confined to a 4-dim. "brane".



Aside on KK-theory: Toy model: $M^5 = \mathbb{R}^4 \times S^1$

$$S = \int d^5x \, \frac{1}{2} (\partial_M \varphi)(\partial^M \varphi) \quad ; \quad M = 0, \dots, 3, 5$$

vac. solution: $\varphi \equiv 0$; expand around this

rename: $x^5 \to y$

Ansatz: $\varphi(x,y) = \sum_{n=0}^{\infty} \varphi_n^c(x) \cos(ny/R) + \sum_{n=1}^{\infty} \varphi_n^s(x) \sin(ny/R)$

$$\Rightarrow \quad S = \frac{L}{2} \int d^4x \left[(\partial \varphi_0^c)^2 + \frac{1}{2} \sum_{n=1}^{\infty} \left\{ (\partial \varphi_n^c)^2 + m_n^2 (\varphi_n^c)^2 + \dots \right\} \right]$$

just ∂_μ ; $\mu = 0 \dots 3$ $\qquad m_n = \frac{n}{R}$

(simple problem: derive this!)

$\Rightarrow$ 4d theory of $\underbrace{\text{zero-mode}}$ + KK modes

Further aside: In this specific case, our zero-mode is a "modulus", i.e., it parametrizes the degeneracy of the vac. solution ($\varphi \equiv c$ is OK for any c, not just zero)

- Back to ADD: SM particles on brane (4d); gravity in (4+n) dimens.

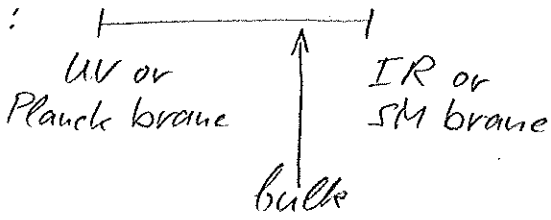
$$\int d^{4+n}x \, M_{P,4+n}^{2+n} R_{(4+n)} \rightarrow \int d^4x \, \underbrace{M_{P,4+n}^{2+n} L^n}_{\equiv M_{P,4}^2} R_{(4)}$$

- Can make $M_{P,4}$ large while keeping $M_{P,4+n} \sim \Lambda \sim$ TeV (by making L large). This nicely solves ② (& ①, if L can be dynamically stabilized). Problem ③ is not addressed

[Problem: Convince yourself that $n = 1$ is ruled out & $n = 2$ is only marginally OK (by calculating the required L). Why do we need to put $\mathcal{L}_{SM}$ on this strange brane?]

5.4 Warped extra dims. ("RS" or "RS I")

- $d = 5$; Comp. space = interval:



$$ds^2 = \underbrace{e^{-2ky}}_{\text{"warp factor"}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

This metric solves Einstein eqs. if $\lambda_{5d} < 0$.
(k determ. in terms of λ_{5d} & $M_{P,5}$. $\lambda_{4d,UV} = -\lambda_{4d,IR} > 0$; both must be tuned rel. to λ_{5d}.)

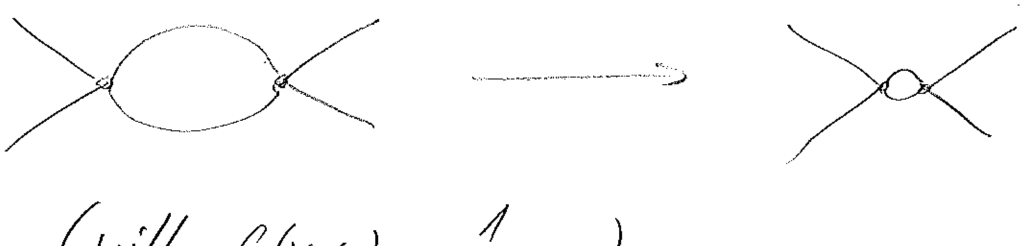
- Smallness of m_{EW} follows from suppr. of IR-brane metric by $\exp[-2k(y_{IR} - y_{UV})]$. $\Rightarrow$ (1) nicely solved since this eff. is exponential.
- (2) can be solved as in ADD.
- (3) naively not solved, but more elaborate modern versions (with some of SM particles in bulk) also address (3), about as successfully as SUSY.

Note: "RS" is related to "technicolor" via AdS/CFT.

6 Strings

Lit. $\to$ cf. "String phenom. notes"

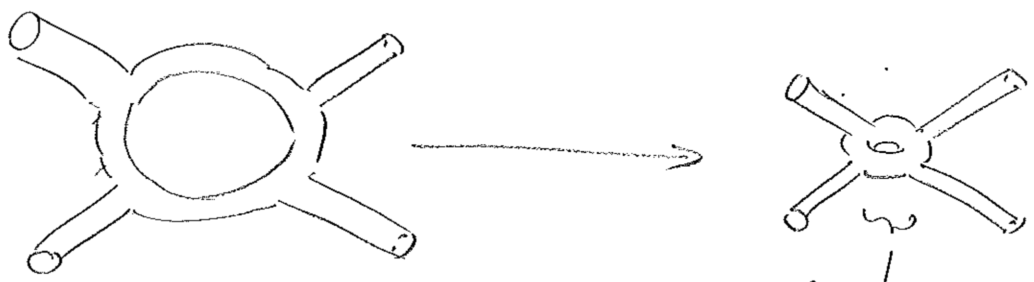
- QFT - divergences:



(with $G(x-y) \sim \frac{1}{(x-y)^2}$)

- The usual renorm. procedure does not work for gravity

- In ST, the problem is solved by:

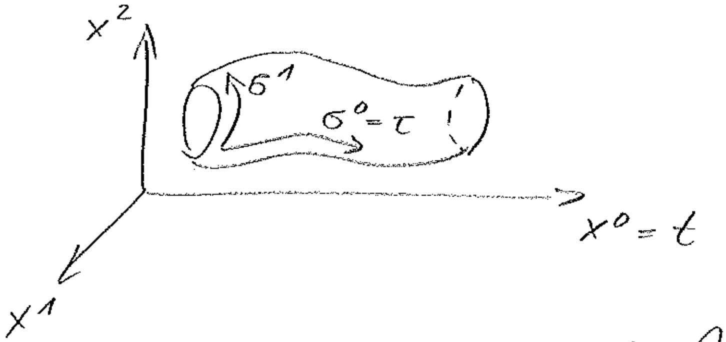


loop "cannot shrink" to size $< l_s$

$\Rightarrow$ no UV div. problem

[Cf. also other approaches: LQG; UV-safety; lattice – but all still have fund. problems & no implic. for physics are in sight.]

- ST-dynamics in more detail:



$$\Rightarrow 2d \text{ QFT with } S \sim \frac{1}{l_s^2}\int d^2\sigma \sqrt{-h}\, \underbrace{h^{\alpha\beta}}_{\text{worldsheet (WS) metric}} (\partial_\alpha X^\mu)(\partial_\beta X^\nu)\, \underbrace{g_{\mu\nu}}_{\text{target space (TS) metric}}$$

$\mu, \nu = 0, 1, \dots, D-1$

[This "Polyakov action" is classically equivalent to the "Nambu-Goto action" $S \sim$ surface area$/l_s^2$]

- D-dim. TS-particle $\equiv$ States of this 2d QFT (in fact: CFT) on $\mathbb{R}\times S^1$ (i.e. vacuum, 1st excitation, 2nd ...)
- $\underbrace{\text{D-dim. Poinc.-symm.}}_{\text{anomaly-free}}$ = internal symm. of WS-theory $\Longrightarrow D = 26$

☺ UV-finite; Graviton (+ other fields) in TS

☹ no fermions; Tachyon (state with $m^2 < 0$)

↑

<u>Way out:</u> Supersymmetrize WS theory: $X^\mu \leftrightarrow \psi^\mu$

anomaly-free $\Rightarrow$ $D = 10$, but now <u>several</u> consistent possibilities (related to amount of SUSY & periodicity conds. of fermions on S^1)

$\Rightarrow$ type I, IIA, IIB, heterotic $SO(32)$, heterotic $E_8 \times E_8$ (all linked by "dualities"; different pert. limits of <u>one</u> theory: "M theory")

<u>Here:</u> IIB

$$S \sim \frac{1}{\ell_s^8} \int d^{10}x \sqrt{-g} \left\{ e^{-2\varphi}\left(R + 4(\partial\varphi)^2 - \frac{1}{2} H_3^2\right) - \frac{1}{2}\left(F_1^2 + F_3^2 + F_5^2\right) \underbrace{+ \cdots}_{\text{CS-terms \& fermions}} \right\}$$

$F_1 = dC_0$ — $(F_1)_\mu = \partial_\mu C_0$

$F_3 = dC_2$ — $(F_3)_{\mu\nu\rho} \sim \underbrace{\partial_\mu (C_2)_{\nu\rho} + \cdots}_{\text{antisymm.}}$

$F_5 = dC_5$

$H_3 = dB_2$

- Put this theory on $M_{10} = \mathbb{R}^4 \times CY$

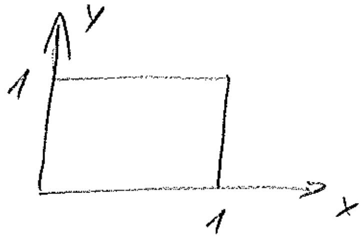
  ↑ very special 6-mf. since $R_{\mu\nu} = 0$ ("Ricci flat") & vac. Einstein-eqs. are hence solved.

- KK-reduction $\Rightarrow$ $\gtrsim 10^4$ 4d eff. th.s. (since # of CY's $\gtrsim 10^4$)

- Actually, the # of possibilities is ~~much~~ larger, due to the option of choosing $\langle F_p \rangle \neq 0$ ("flux")

7) Flux quantization in a nutshell

- Toy model: U_1 gauge th. on $M_6 = \mathbb{R}^4 \times T^2$
- Focus on T^2:



$$\varphi(x+n, y+m) = \varphi(x,y)$$

$\forall\, n, m \in \mathbb{Z}$

$\forall$ fields

- Consider, more specifically, A_M $\quad (M \in \{0, \ldots, 3, \underset{\downarrow\atop x}{5}, \underset{\downarrow\atop y}{6}\})$

Let $A_6 \equiv A_y = \alpha x \;\Rightarrow\; F_{56} \equiv F_{xy} = \partial_x A_y = \alpha$

(all other A's zero)

- Periodicity: in y-direction – trivial
  in x-direction – $A_y(x+1, y) \overset{!}{=} A_y(x,y)$

$$\alpha x + \alpha \overset{!}{=} \alpha x$$

- For this to work with $\alpha \neq 0$, need to also use gauge trf.:

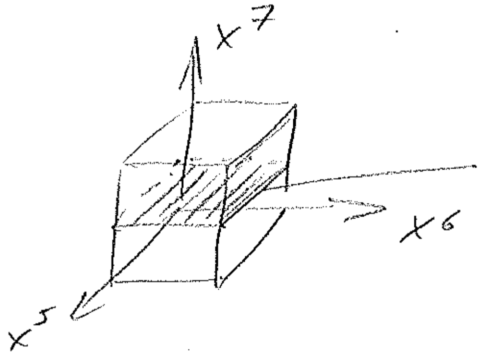
$$A_y \to A_y + \partial_y \chi$$

need χ with $\partial_y \chi = \alpha \;\Rightarrow\; \chi = \alpha y$

- But, in case there are charged fields ψ, we also have
  $\psi \to e^{i\alpha y}\psi$
- For ψ periodic, $e^{i\alpha y}\psi$ is only periodic if $\alpha = 2\pi n$
  $(n \in \mathbb{Z})$
- The flux F_{56} (actually $\int_{T^2} F$) is quantized. We can choose n at will. This field conf. <u>can not</u> decay to vacuum.

(Problem: Perform the same analysis (at least qualitatively) for S^2 and relate it to what you learned about the Dirac monopole.)

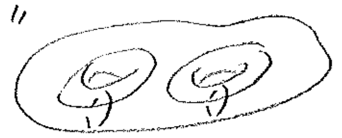
- Now consider 7d U_1 gauge th. on T^3:



T^2 embedded in T^3 (there are actually 3 diff. possibilities for such a $T^2 \subset T^3$)

$\Downarrow$

T^3 has 3 different "2-cycles".

- Analogous: "  " has 4 diff. 1-cycles.

- Each p-cycle of comp. space can carry n units of p-form-flux ($\langle F_p \rangle \neq 0$).

- Relevant for our type IIB-case: typical CY's have 100reds of 3-cycles; each can carry F_3 & H_3 flux.

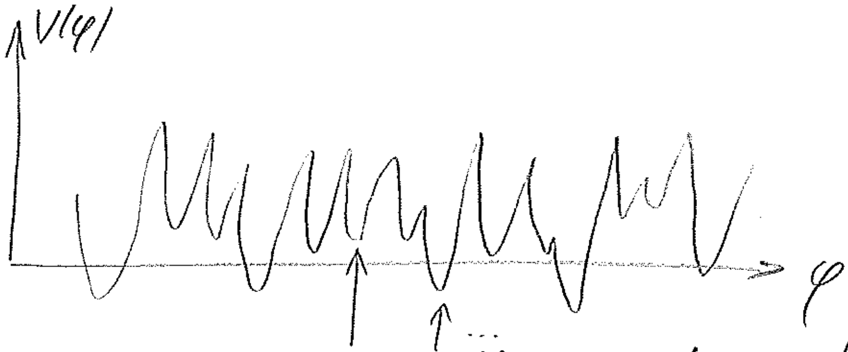
8 Landscape

Take e.g. some CY with 200 3-cycles, let $n \in \{-10 \dots 10\}$

$$\Rightarrow \# \text{ of choices} = 20^{400} \sim \underbrace{10^{500}}_{\# \text{ of diff. 4d models}}$$

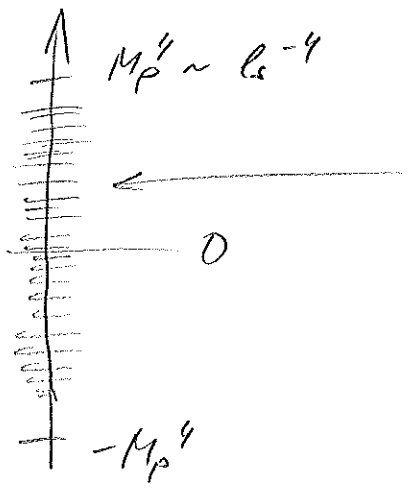
- There exist processes linking those solutions, but for this we need to violate class. EOMs.

- Illustrative example: 4d QFT with special $V(\varphi)$:



many diff. soln. of <u>one</u> theory
(from the low-energy persp., each minimum corresp. to a diff. part. phys. model since, e.g. m_φ & λ vary from "vacuum" to "vacuum"; in the "real" ST landscape, also particle content, gauge groups, Yukawas etc. vary)

- <u>Crucial:</u> Distribution of λ's:



reasonably uniform distrib. with <u>no</u> particularly strong suppression or enhancement near zero

(cf. papers by Denef/Douglas)

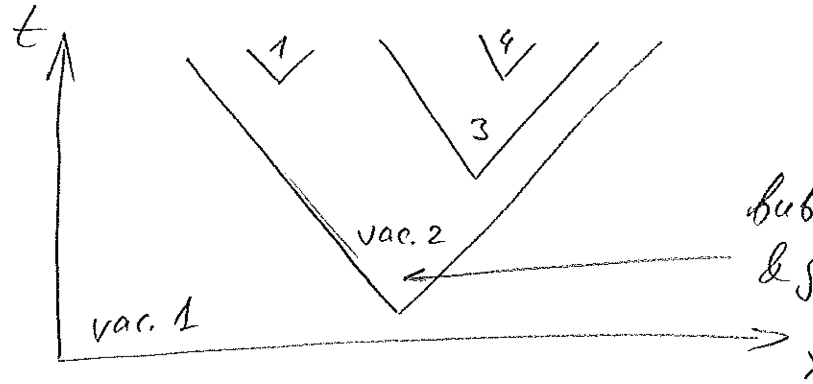
- Obviously, if typically $\lambda \sim O(M_P^4)$, but we have 10^{500} random examples as above, we expect to find some models with λ as low as $10^{-500} M_P^4$.
Since $\lambda_{obs.} \sim 10^{-120} M_P^4$, we are statistically guaranteed to find compactifications with sufficiently small λ!

$\Rightarrow \lambda_{obs.}$ is not a problem for ST if we allow this type of "fine-tuning in the ST landscape"

<u>Thus:</u> ST is a serious candidate for describing the <u>real</u> world, including quant. gravity, λ-fine-tuning (and maybe many further fine-tunings, e.g. m_h^2)

- But why should we live in such an "unlikely" vacuum; how did we get there?

- <u>Idea</u>: <u>eternal inflation</u>



bubble of <u>new</u> vac. nucleates & grows with speed-of-light in <u>old</u> vacuum.

(only works for $\lambda > 0$. If $\lambda < 0$, we face a "big crunch". But still – this way the whole landscape gets "populated")

- We live "where we can live", i.e. where $|\lambda|$ is small enough such that e.g. galaxies can form.
(cf. Weinbergs (anthropic) prediction of the right order-of-magn. of λ)

- One would like to go beyond this and make more (statistical) predictions in this "<u>multiverse</u>". But the "<u>measure problem</u>" of the landscape appears to be a very hard problem indeed...

$\boxed{\text{Continuation:}}$ $\curvearrowright$ "String Phenom." notes

Problem 1: SU_5 GUT

a) Normalize $T_1 \equiv T_{U_1}$ according to $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$.

b) Derive $\bar{5} = (\bar{3},1)_2 + (1,2)_{-3}$ (The only non-trivial point is $2 \sim \bar{2}$ for SU_2)

$$3\cdot 2^2 + 2\cdot 3^2 = 30 \Rightarrow T_1 = \frac{1}{\sqrt{60}}\begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}$$

$$\begin{tikzcd} \psi^i \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{comm.}"] & U^i{}_j \psi^j \arrow[d] \\ \varepsilon_{ij}\psi^j \arrow[r] & U^*_i{}^k \varepsilon_{kj}\psi^j = \varepsilon_{ik} U^k{}_j \psi^j \end{tikzcd}$$

This needs to be shown.

Mult. both sides by $U^i{}_l$.

Use $U^i{}_l U^*_i{}^k = (U^\dagger U)^k{}_l = \delta^k{}_l$

Use $U^i{}_l \varepsilon_{ik} U^k{}_j = \varepsilon_{ik} U^i{}_l U^k{}_j = \varepsilon_{lj} \det U$
$= \varepsilon_{lj}$ ✓

c) $T_1 = \gamma \cdot Y$; determine γ
(this is crucial since it is not g_1 but g_1/γ which unifies with g_2 & g_3!)

$Y(d) = 1/3$; $T_1(d) = -\left(\frac{1}{\sqrt{60}}\cdot(-2)\right) = \frac{1}{\sqrt{15}}$

from compl. conj.

$$\gamma = \frac{T_1(d)}{Y(d)} = \frac{3}{\sqrt{15}} = \sqrt{\frac{3}{5}}$$ ... famous factor!

d) Derive the branching rule of the "10"!

Simple part: $\psi^{ij} \to U^i{}_k U^j{}_l \psi^{kl}$

$\left(\begin{array}{c|c} SU_3 & \\ \hline & SU_2 \end{array}\right)$; $\quad 10 = (3\times 3_A, 1) + (3,2) + (1, \underbrace{2\times 2_A}_{=1})$

T_1-charges: $-2-2=-4$
$-2+3=1$
$+3+3=6$

Only non-trivial part: $3\times 3_A \sim \bar{3}$

$$\begin{array}{ccc} \psi^{ij} & \longrightarrow & U^i{}_k U^j{}_l \psi^{kl} \\ \downarrow & & \downarrow \\ \varepsilon_{ijk}\psi^{jk} & \longrightarrow & (U^*)_i{}^l \varepsilon_{ljk}\psi^{jk} = \varepsilon_{imn} U^m{}_k U^n{}_l \psi^{kl} \end{array}$$

... same det-trick as in SU_2-case works

Problem 2: No-scale models

Recall that $V = e^K \left(K^{\alpha\bar\beta} D_\alpha W \overline{(D_\beta W)} - 3|W|^2 \right)$.

a) Show that the "no-scale property" $\boxed{K^{\alpha\bar\beta} K_\alpha K_{\bar\beta} = 3}$ together with $W = W_0 = \text{const.}$ implies $V \equiv 0$.

b) Show that $K = -3 \ln (T + \bar T)$ has the no-scale property.

c) Derive the no-scale property for any K for which e^K is a homog. fct. of degree -3 in the variables $(\tau^\alpha + \bar\tau^{\bar\alpha})$ [with $\alpha = 1 \cdots n$].

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a) $D_\alpha W = \partial_\alpha W_0 + K_\alpha W_0 = K_\alpha W_0$; $\overline{K_\beta} = K_{\bar\beta}$

the rest is obvious

K is real. Without loss of generality let
$K = K_0 + c\tau + \bar c \bar\tau$
$\overline{K_\tau} = \overline{(c\tau)_\tau} = \bar c = K_{\bar\tau}$

---

b) $K_T = -\frac{3}{T + \bar T} = K_{\bar T}$; $K_{T\bar T} = \frac{3}{(T + \bar T)^2}$; $K^{T\bar T} = \frac{(T + \bar T)^2}{3}$

rest obvious

---

c) Let $F \equiv e^K$. Euler: $F_\alpha \tau^\alpha + F_{\bar\alpha} \bar\tau^{\bar\alpha} = -3F$

Since F is fct. of $(\tau^\alpha + \bar\tau^{\bar\alpha})$, $F_\alpha = F_{\bar\alpha}$.

$\Rightarrow F_\alpha \cdot (\tau^\alpha + \bar\tau^{\bar\alpha}) = -3F$; $(\ln F)_\alpha \cdot (\tau^\alpha + \bar\tau^{\bar\alpha}) = -3$

$$K_\alpha \cdot (\tau^\alpha + \bar{\tau}^{\bar{\alpha}}) = -3$$

$$\Big\downarrow \ \frac{\partial}{\partial \bar{\tau}^{\bar{\beta}}}$$

$$K_{\alpha\bar{\beta}} (\tau^\alpha + \bar{\tau}^{\bar{\alpha}}) + K_{\bar{\beta}} = 0$$

$$\Big\downarrow \ \cdot K^{\gamma\bar{\beta}}$$

$$(\tau^\gamma + \bar{\tau}^{\bar{\gamma}}) + K^\gamma = 0$$

plug in above

$$\Downarrow$$

$$K_\alpha K^\alpha = 3 \quad \checkmark$$

P5

$$\partial_T e^K W\bar{W} = \left(K_T + \frac{W_T}{W}\right) e^K W\bar{W} = (D_T W) e^K \bar{W} = 0$$

Prove that $V(T,\bar{T})$ has extremum at point of $D_T W = 0$.
(i.e. at point of unbroken SUSY).

(Param. analyse KKLT)

$$K = -3\ln(T+\bar{T}) \; ; \quad W = W_0 + e^{-T} \; ; \qquad K_T = -\frac{3}{T+\bar{T}}$$

$$K_{T\bar{T}} = \frac{3}{(T+\bar{T})^2}$$

$$e^K \left(K_{T\bar{T}}^{-1} |D_T W|^2 - 3|W|^2 \right)$$

$$D_T W = -e^{-T} - \frac{3}{T+\bar{T}} (W_0 + e^{-T})$$

$$= \frac{1}{(T+\bar{T})^3} \left[\frac{(T+\bar{T})^2}{3} \left| -e^{-T} - \frac{3}{T+\bar{T}} (W_0 + e^{-T}) \right|^2 - 3|W_0 + e^{-T}|^2 \right]$$

$$= \frac{1}{(T+\bar{T})^3} \left[\frac{(T+\bar{T})^2}{3} \left\{ e^{-T+\bar{T}} + \frac{e^{-\bar{T}}}{T+\bar{T}} (W_0 + e^{-T}) + \text{h.c.} \right\} \right]$$

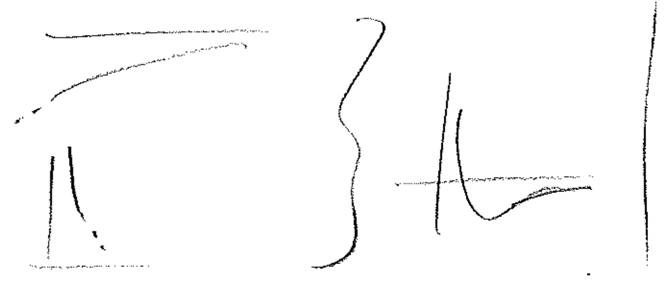
$$= \frac{1}{3(T+\bar{T})} \left\{ e^{-T+\bar{T}} + \frac{2e^{-T+\bar{T}}}{T+\bar{T}} + \frac{e^{-\bar{T}} W_0 + e^{-T} \bar{W}_0}{T+\bar{T}} \right\}$$

$$T+\bar{T} = 2t$$

$$= \frac{1}{6t} \left\{ e^{-2t} \left(1 + \frac{1}{t}\right) - |W_0| \frac{e^{-t}}{t} \right\}$$

$t \to \infty$:

$t \to 0$:



$|W_0| \ll 1$