De Sitter from String Theory: Control Issues of KKLT

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original part based mostly on work with **Xin Gao and Daniel Junghans**

(includes also comments on earlier work with **Hamada/Shiu/Soler**)

Outline

- The difficulty of realizing de Sitter in string theory.
- KKLT and and some of its potential problem.
- The Singular-Bulk Problem of KKLT.

String Compactifications

- String theory provides an (essentially unique) and UV-complete field theory in 10d:

$$S = \int_{10} \mathcal{R} - |F_{\mu\nu\rho}|^2 + \cdots$$

- At the very least, this is a useful toy-model for a well-defined gravitational theory.

- One may go for more by compactifying on Calabi-Yaus (6d spaces with vanishing Ricci tensor).

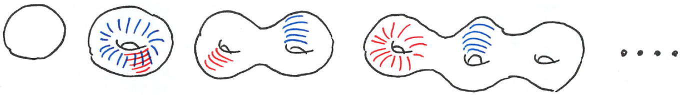
- One ends up with

  (A) unrealistic moduli-space field theories ($\mathcal{N} = 2$ SUSY)

  (B) very flat and poorly controlles field spaces ($\mathcal{N} = 1$ SUSY) [it remains unclear how $\Lambda \sim 10^{-120}$ can occur].

String compactifications: flux landscape

- The extra ingredient of fluxes induces an exponentially large landscape of discrete solutions.



Bousso/Polchinski '00, Giddings/Kachru/Polchinski '01 (GKP)
Kachru/Kallosh/Linde/Trivedi '03 (KKLT), Denef/Douglas '04
Balasubramanian/Berglund/Conlon/Quevedo '05 (LVS)

- Key to the historical number 10^{500} (by now rather $10^{300.000}$) is not the abundance of Calabi-Yaus ($\sim 10^9$), but the discrete flux choice:

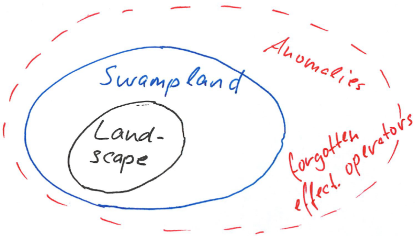
$$\oint_{3-cycle} F_{\mu\nu\rho} \in \mathbb{Z}$$

Landscape vs. Swampland

- Given this abundance of solutions, one must wonder whether 'anything goes' in string compactification.

- This leads to the Landscape/Swampland program:

  Vafa' 05; Ooguri/Vafa '06



- I will not discuss the many interesting aspects of this (no global symmetries, field-excursions, weak gravity, ...) but focus on de Sitter:

De Sitter swampland conjectures

- One possible constraint is clearly $\Lambda_{cosm.} \leq 0$.
- Indeed, a longstanding unease about the status of de Sitter space in quantum gravity exists.

Woodard, Danielsson, Van Riet, Bena, Grana, Sethi, Dvali, ...

- More recently, concrete formulations of varying strength have been considered within the Swampland program (e.g. $V'/V > \mathcal{O}(1)$ or $V''/V < -\mathcal{O}(1)$)

Danielsson/Van Riet
Obied/Ooguri/Spodyneiko/Vafa
Garg/Krishnan, Andriot
Ooguri/Palti/Shiu/Vafa '18

(see also further related work by Andriot, Cirbiori et al. ...)

Problems with de Sitter in string compactifications

- Let us briefly pause to explain one of the reasons why realizing de Sitter is difficult.

- The generic result of a compactification with volume $\mathcal{V}$ (and some positive-energy source in the compact space) is

$$\mathcal{L} \sim \mathcal{V}\left[\mathcal{R}_4 - \frac{(\partial\mathcal{V})^2}{\mathcal{V}^2} - E\right] .$$

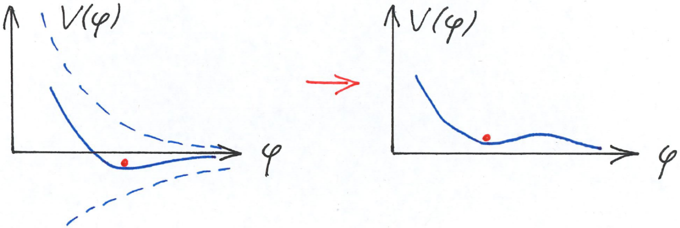
- After Weyl-rescaling to the Einstein frame and introducing the canonical field $\varphi = \ln(\mathcal{V})$, one finds

$$\mathcal{L} \sim \left[\mathcal{R}_4 - (\partial\varphi)^2 - E\, e^{-\varphi}\right] .$$

- The exponent is usually $\mathcal{O}(1)$, so the simplest compactifications lead to steep potentials: $|V'|/V \sim \mathcal{O}(1)$.

String compactifications: flux landscape

- Combining two such runaway potentials with different sign allows in principle for AdS solutions.
- At least 3 potential terms with different falloff and appropriate coefficients are needed to get dS.



If all parameters involved are $\mathcal{O}(1)$, this can never happen in parametric control.

Dine/Seiberg '85
Ooguri/Palti/Shiu/Vafa '18

However, with some tuning of fluxes effective small and large parameters can be realized.

The earliest such scenario for realizing dS was

KKLT

Kachru/Kallosh/Linde/Trivedi '03

An alternative is the 'large volume scenario' or LVS

Balasubramanian/Berglund/Conlon/Quevedo '05

We will first recall how KKLT works and discuss recent criticism by

Moritz/Retolaza/Westphal '17

which was historically important in the above debate.

But then we will come to a rather different concern, which at the moment appears to threaten KKLT more seriously ....

(2-slide reminder of) KKLT

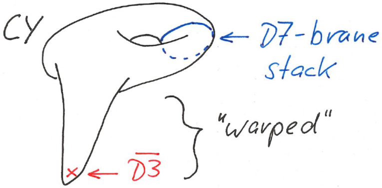
- CY with all complex-structure moduli fixed by fluxes;
  The only field left: Kahler modulus $T = \tau + ic$ with $\tau \sim \mathcal{V}^{2/3}$.

- $K = -3\ln(T + \overline{T})$;   fluxes give $W = W_0 =$ const.,

  $\Rightarrow \quad V \equiv 0 \quad$ ('no scale') .

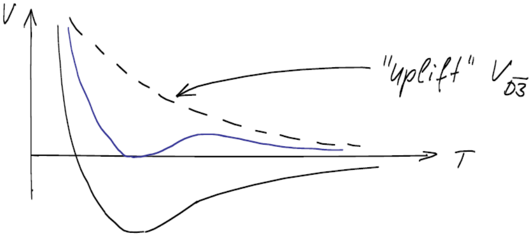
- Gaugino condensation on D7 brane stack: $W = W_0 + e^{-T}$.

- Small uplift by $\overline{\text{D3}}$-brane
  in a warped throat:
  $V \rightarrow V + c/\tau^2$.



KKLT

- The scalar potential is changed first to SUSY-AdS, then to an 'uplifted' meta-stable de Sitter potential:



- A longstanding critical debate has targeted the metastability of the $\overline{D3}$ in view of flux-backreaction.

  (My take on this is that metastability remains plausible.)

Bena, Grana, Danielsson, Van Riet, ...

KKLT under attack

Moritz/Retolaza/Westphal '17
Gautason/Van Hemelryck/Van Riet '18

- Recent criticism was rooted in a possibly too simplistic treatment of D7-gaugino–bulk-coupling:

$$\mathcal{L}_{10} \supset |G_3|^2 + G_3 \cdot \Omega_3 \langle \lambda\lambda \rangle \delta_{D7} \, .$$

Camara/Ibanez/Uranga '04, Koerber/Martucci '07
Baumann/Dymarsky/Klebanov/Maldacena/McAllister '06
Heidenreich/McAllister/Torroba '10

- It is clear what to expect:
  G_3 backreacts, becoming itself singular at the brane.

- Plugging this back into the action,
  one gets a divergent effect of type $(\delta_{D7})^2$.

- Now anything can happen....

KKLT rescued

Hamada/AH/Shiu/Soler '18,'19; Kallosh '19; Carta/Moritz/Westphal '19

- Singular gaugino effects have been observed before, in other string models.

  Horava/Witten '96

- It has been shown that a highly singular $\langle\lambda\lambda\rangle^2$-term saves the day by 'completing the square'. Applied to our case:

$$\mathcal{L}_{10} \supset \left| G_3 + \Omega_3 \langle\lambda\lambda\rangle \, \delta_{D7} \right|^2 .$$

- Very roughly speaking, one now writes $G_3 = G_3^{flux} + \delta G_3$ and lets the second term cancel (most of) the δ-function.

  The result is (**very** roughly):

$$\mathcal{L}_{10} \supset \left| G_3^{flux} + \langle\lambda\lambda\rangle \right|^2 \quad \rightarrow \quad \left| D_T W_0 + \partial_T e^{-T} \right|^2 .$$

The perfect square structure in M-theory

- The established part of the story is in M-theory (with x^{11} compactified on $S^1/\mathbb{Z}_2$). There, one has

$$S \sim -\int_{11} \left(G_4^2 \;-\; \delta(x^{11})(G_4)_{ABC\,11}\, j^{ABC} \right),$$

where $j^{ABC} \sim \overline{\lambda}\,\Gamma^{ABC}\lambda$.

- It is well-known that the divergence problem is resolved by the proposal (enforced by SUSY)

Horava/Witten

$$S \sim -\int_{11} \left(G_4 - \frac{1}{2}\delta(x^{11})\, j \right)^2 .$$

- Our proposal basically describes how an analogous quartic gaugino term on the brane must be added in type IIB.

(cf. Hamada/AH/Shiu/Soler '18/'19 for details)

In summary:

10d perfect square structure leads to
4d SUGRA perfect square structure
and to KKLT, including possible uplift.

$$e^K K^{T\overline{T}} \Big| D_T(W_0 + e^{-T}) \Big|^2$$

Recent related work by other groups

agreement with Carta/Moritz/Westphal,
still (partial) disagreement with Gautason/Van Hemelryck/Van Riet/Venken

Using Generalized Complex Geometry, the AdS parameter can be related to a parameter in 10d SUSY conditions.
$\Rightarrow$ fully 10d-local check of pre-uplift KKLT

Bena/Grana/Kovensky/Retolaza

Related attempt of component-level check w/o SUSY:

Kachru/Kim/McAllister/Zimet

However, non-local D7 action introduced ad hoc;
divergence cancellation in G_3 kinetic term remains unclear.

The advertise new concern starts with the

The Throat Glueing Problem

Carta/Moritz/Westphal '19

- Recall basic parametrics of KKLT:

$$V_{AdS} \sim -e^{-4\pi \mathrm{Re}(T)} \qquad \text{vs.} \qquad V_{Uplift} \sim e^{-8\pi K/3g_s M}.$$

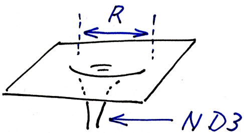
(Here K and M are the flux numbers of the two 3-cycles of the KS throat.)

- For a metastable uplift to dS, the two potentials must match:

$$\Rightarrow \qquad \mathrm{Re}(T) \simeq 2K/3g_s M.$$

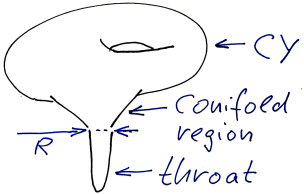
- At the same time, the throat carries $N = KM$ units of D3 charge, giving it a radius

$$R^4_{throat} \simeq g_s N.$$



Throat Glueing Problem (continued)

- However, at least most naively, $g_s \, \mathrm{Re}(T) \sim R^4_{CY}$ and the standard picture



implies $R^4_{throat} < R^4_{CY}$.

- With the previous estimates, this leads to the problematic inequality

$$g_s N \lesssim K/M$$

or (using $K = N/M$)

$$\mathcal{O}(1) \lesssim 1/g_s M^2 \,.$$

Throat Glueing Problem (continued)

- The problem is that $g_s M \simeq R^2_{S^3} \gtrsim 1$

  KS, KPV, Klebanov/Herzog/Ouyang '01

  for supergravity control and $M \gtrsim 12$

  KPV (see also Bena/Dudas/Grana/Lüst, Blumenhagen/Kläwer/Schlechter)

  for metastability of the anti-D3-brane.

- Thus, the standard picture of a small throat glued into the large bulk of a CY can not be maintained.

  (See App. of our paper for the (2π)-factors etc.
  It turns out these do not resolve the problem $g_s M^2 \lesssim 1$, which will remain central throughout the talk.)

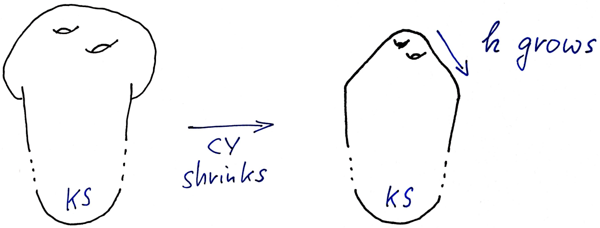
Is the Throat Glueing Problem deadly ?

- Not obviously, since a priori the warp factor $h(y)$ of

$$ds_{10}^2 = h(y)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h(y)^{1/2}\tilde{g}_{mn}dy^m dy^n$$

is just some function on the CY.

- The Kahler modulus corresponds to $h(y) \to h(y) + \text{const.}$ It is a flat direction 'at the level of GKP'. So we may simply make the bulk smaller than the throat!



The singular-bulk problem

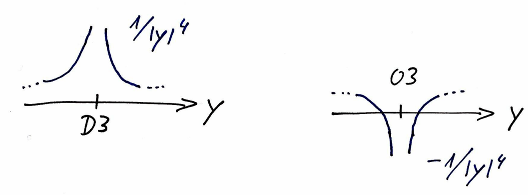
- An actual problem is not that the geometry defies our standard intuition, it is that the CY may be forced into a singular regime, since $h < 0$.
- The danger of growing singualrities as $h \to h - \text{const.}$ has already been discussed in the Appednix of Carta et a;., but without turning this into a quantitative problem for KKLT.
- The goal of the rest of the talk is exactly this:

  Demonstrate that, generically, the regime of KKLT is enforcing $\mathbf{h < 0}$ in a large portion of the CY geometry.

The singular-bulk problem (continued)

- Before starting, let us recall the standard behavior of h near D3-branes/O3-planes:

$h(y)$:



- The string-sized negative regions near O3s are not a problem
- Also having many O3s is a priori not a problem as long they are scattered, each with it's small negative region.
- The bulk singularity problem arises from the 'macroscopic' behaviour of $h(y)$.

The singular-bulk problem (continued)

- For quantifying the problem, a key insight is that the warped E3 size $\mathcal{V}_\Sigma$ determines the exponential effect:

$$\mathrm{Re}(T) \sim N/g_s M^2 \qquad \Rightarrow \qquad \mathcal{V}_\Sigma \sim N/M^2$$

with

$$\mathcal{V}_\Sigma = \int_\Sigma \sqrt{\tilde{g}}\, h(y) = \tilde{\mathcal{V}}_\Sigma \langle\, h \,\rangle_\Sigma \,.$$

- W.l.o.g., we use a CY such that $\tilde{\mathcal{V}} = \int_{CY} \sqrt{\tilde{g}} = 1$.
Hence $\tilde{\mathcal{V}}_\Sigma$ is an $\mathcal{O}(1)$ number.
$\Rightarrow$ We are constraining the warp factor on Σ:

$$\langle\, h \,\rangle_\Sigma \sim N/M^2 \tilde{\mathcal{V}}_\Sigma \sim N/M^2 \,.$$

The singular-bulk problem (continued)

- In summary, for a large part of the E3 locus Σ we have

$$h \lesssim N/M^2 \,.$$

- We also know from GKP that h represent a form of 'electrostatic potential' for the D3 charge density on the CY:

$$-\tilde{\nabla}^2 h \,=\, g_s \,\tilde{\rho}_{D3} \,.$$

  Our normalization is such that $\tilde{\rho}_{D3}$ is a CY-metric δ-function for a single D3 brane.

- We see that h is a compact-space Green's function for a charge distribution of

  $g_s N$ units of positive charge, localized at conifold

  $-g_s N$ units of negative charge, scattered in the CY.

The singular-bulk problem (continued)

- If the parameter $g_s N$ were $\mathcal{O}(1)$, we would have $|\tilde{\partial} h| \sim 1$.
  (The details of the function are fixed by geometry and charge distribution. An additive constant is undetermined.)

- But in our case the variation is scaled up by $g_s N \gg 1$.
  At the same time h is bounded on the E3: $h \lesssim N/M^2$.

$$\Rightarrow \qquad \boxed{\frac{|\tilde{\partial} h|}{h} \gtrsim g_s M^2 \gtrsim M \gg 1}$$

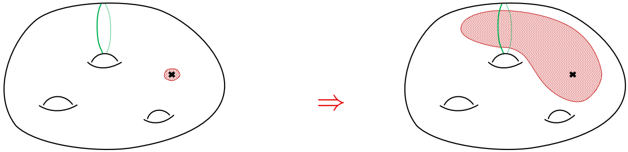
Now, by Taylor expanding at a point y_0 of the E3,

$$h(y_0 + \delta y) \approx h(y_0) + \partial_m h(y_0)\, \delta y^m \,,$$

we see that h runs negative near the E3: $|\tilde{\delta y}| \lesssim 1/g_s M^2$.

The singular-bulk problem (continued)

- The argument also works if δy is a brane-parallel direction, making much of the E3 singular:



- Alternative view of the problem:

$$R_6 = h^{-5/2}|\tilde{\partial} h|^2 - \frac{3}{2} h^{-3/2} \tilde{\nabla}^2 h \qquad \Rightarrow \qquad R_6 \gtrsim g_s^2 M^5/\sqrt{N}$$

Imposing $g_s M \gtrsim 1$, $M \gtrsim 12$ and $R_6 \lesssim 1$ implies $N \gtrsim 3 \cdot 10^6$.

This exceeds the largest know tadpole of $7 \cdot 10^4$.

Taylor/Wang '15

Singular-bulk problem with coarse-grained warp factor

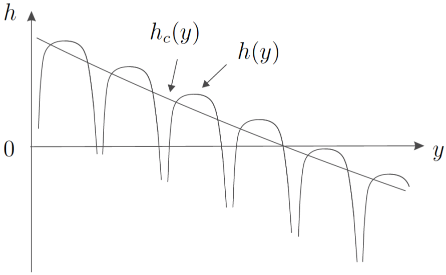
- One may think in terms of a coarse-grained warp factor (cf. the coarse-grained electrostatic potential in a plasma).

- For example:

$$h_c(y) = \frac{\int d^6y'\, h(y')\, \exp(-|y - y'|^2/d^2)}{\int d^6y'\, \exp(-|y - y'|^2/d^2)}$$

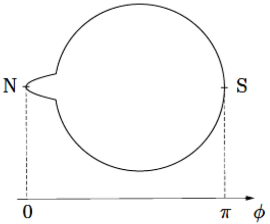
- One can show that h_c closely follows the maxima of h.

- It becomes apparent that even h_c goes negative, so the problem is distinct from O3-singularities



Singular-bulk problem in a toy model

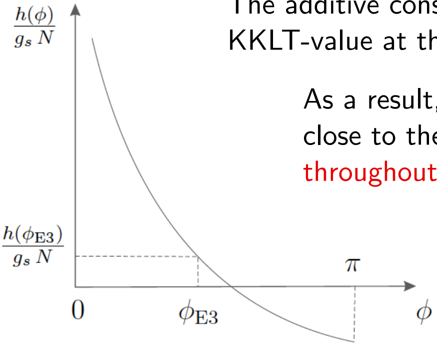
- To develop some intuition, let us consider a simple toy model.
- Replace the CY by an S^6, with the throat at the north pole.
- Let the (O3-plane) negative charge be scattered/smeared homogeneously.



- The E3 will be modelled as an S^4 positioned at some fixed altitude φ.

Singular-bulk problem in a toy model (continued)

- $h(\varphi)$ is naturally very large near the north pole.
- It has some smooth, non-constant behaviour in the bulk.



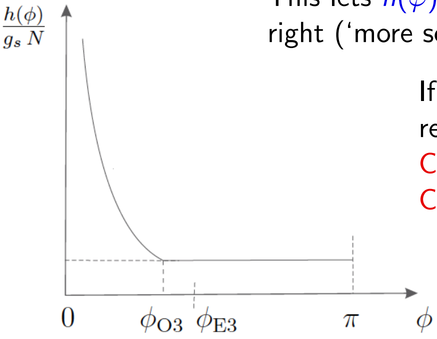
The additive constant is adjusted to ensure the KKLT-value at the E3 position.

As a result, $h(\varphi)$ goes negative close to the E3 and stays negative throughout the south pole region.

Singular-bulk problem in a toy model (continued)

- Moving the E3 to the south pole ($\varphi = \pi$) is presumably only possible in the toy model since the E3 cycle is trivial.

- A more generally useful option may be the cancellation of the tadpole close to the throat.



This lets $h(\varphi)$ be constant everywhere to the right ('more south') of the O3-location

If the E3 locus is in this constant-h region, everything may be fine.
Clearly, this requires very special CY-orientifolds!

Escape routes

- One option, suggested by the toy model, is a very special arrangement of the O3s (or the curved O7/D7s).
  Very challenging to study this in proper CY geometries!

- Another option is to the observation that the problematic 'small parameter' changes if the E3 is replaced by gaugino condensation:

$$1/g_s M^2 \quad \rightarrow \quad N_c/g_s M^2 \,.$$

- However, $N_c \gg 1$ appears to always come with $h^{1,1} \gg 1$. The latter is problematic, as we will see in a moment.

Louis/Rummel/Valandro/Westphal '12, Carta/Moritz/Westphal '19

Escape routes (continued)

- At first sight, making $h^{1,1}$ large appears promising even before thinking about $N_c \gg 1$.

- The reason is that, if we do not assume $\tilde{\mathcal{V}}_\Sigma \sim 1$, then the problematic small parameter changes as

$$1/g_s M^2 \quad \to \quad 1/g_s M^2 \tilde{\mathcal{V}}_\Sigma \,.$$

  (Recall that $\tilde{\mathcal{V}} = 1$ by convention.)

- This could help since $\tilde{\mathcal{V}}_\Sigma \ll \tilde{\mathcal{V}}$ is the natural expectation in CYs with $h^{1,1} \gg 1$.

- Using volumes measured in string units, one explicitly needs:

$$\tau_\Sigma / \mathcal{V}^{2/3} \lesssim 1/g_s M^2$$

  for all 4-cycles Σ.

Escape routes – problems at large $h^{1,1}$

- However, according to an analysis of a large class of CYs, there is a problem due to the $h^{1,1}$ scaling of various volumes:

  Demirtas/Long/McAllister/Stillman '18

- If the curves are kept large enough for SUGRA control, then surfaces and the volume scale as

$$\tau \sim (h^{1,1})^{3.2\cdots4.3}\,, \quad \mathcal{V} \sim (h^{1,1})^{6.2\cdots7.2} \qquad (h^{1,1} \gg 1)\,.$$

- Combining this with $\tau/\mathcal{V}^{2/3} \lesssim 1/g_s M^2$ and $\tau \sim N/g_s M^2$ gives

$$\boxed{N/g_s M^2 \gtrsim (h^{1,1})^{3.2} \gtrsim (g_s M^2)^{4.8}}$$

- With the familiar bound on $g_s M^2$,
  this enforces $N \gtrsim 2 \cdot 10^6$. Too large!

Escape routes – combining large N_c and large $h^{1,1}$

- Now let us, in addition, use large-N_c gaugino condensation instead of instantons. We accept the empirical relation

$$N_c \sim \beta h^{1,1} \qquad \text{at} \qquad h^{1,1} \gg 1 \quad (\text{and } \beta \sim \mathcal{O}(1)\,)\,.$$

Louis/Rummel/Valandro/Westphal '12

- Then the previous problematic chain of inequalities turns into

$$\boxed{\frac{N\,\beta h^{1,1}}{g_s M^2} \gtrsim (h^{1,1})^{3.2} \gtrsim \left(\frac{g_s M^2}{\beta h^{1,1}}\right)^{4.8}\,.}$$

- The outcome for N changes:

$$N \sim (g_s M^2)^{5.8} \gtrsim 2 \cdot 10^6 \quad \Rightarrow \quad N \sim (g_s M^2/\beta)^{2.3}\,.$$

- Thus, numerically this escape route works. But we have here assumed a 7-brane gauge group with $N_c \sim \beta h^{1,1}$ on every 4-cycle! Is that possible?

Further control issue: Topology too complicated?

- Because $N \gg N/g_s M^2 \sim \tau_\Sigma$, parametric control needs a very large tadpole. In the best-understood cases, this comes with complicated topology

  $\Rightarrow$ Too many 'cycles per volume'.

- In F-theory

$$24N = \chi(Y) = 6(8 + h^{1,1}(Y) + h^{3,1}(Y) - h^{2,1}(Y))\,.$$

Klemm/Lian/Roan/Yau '98

  $\Rightarrow$ Need large $h^{1,1}(Y)$ or large $h^{3,1}(Y)$.

  In the first case, use $h^{1,1}(Y) = h^{1,1}_+(X) + 1$.

- Thus, we consider CYs with $h^{1,1} \sim N$. But this clashes with the previous relation $\tau_\Sigma \ll N$ and $\tau_\Sigma \sim (h^{1,1})^{3.2} \sim N^{3.2}$.

  Demirtas/Long/McAllister/Stillman '18

- The route of large $h^{3,1}(Y)$ also looks complicated but not completely excluded...

Summary / Conclusions

- One should not simply believe that metastable stringy de Sitter is possible/impossible but try to demonstrate it.
- Concerning the recent '10d-line-of-attack', KKLT appears to be in better shape now than two years ago.
- However, it may fall victim to the bulk singularity problem discussed above.
- The escape routes appear complicated and non-generic, but that does not make them hopeless. Also, the LVS does not suffer from this issue.
- In parallel to (dis)proving KKLT/LVS in more and more detail, we should try to get stringy quintessence to work.
- This is not easy....(cf. recent paper on the F-term problem)

An Aside on Quintessence:

- It is conceivable that all dS constructions will fail in the end.
- Quintessence is a natural way out, but this is also difficult..

see e.g. Cicoli/Pedro/Tasinato '12
(also: Cicoli/Burgess/Quevedo '11)

- In particular, one faces an *F*-Term Problem:

AH/Skrzypek/Wittner

- Namely, one needs an extremely large volume, where phenomenological SUSY-breaking implies:

$$e^K |D_x W|^2 \qquad \gg \qquad \left| e^K (|D_T W|^2 - 3|W|^2) \right|$$

$\Rightarrow$ completely new scalar-potential term needed!

Selection of recent work: Cicoli/DeAlwis/Maharana/Muia/Quevedo; Acharya/Maharana/Muia; Emelin/Tatar; Hardy/Parameswaran; $\cdots$