The Weak Gravity Conjecture

and the Axionic Black Hole Paradox

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Outline

- Motivation
- Axionic Black Holes
- Deriving a Weak Gravity Conjecture
  from the need to avoid exotic remnants
- What, if Weak Gravity is violated only in the effective theory?

Motivation

- The Weak Gravity Conjecture,

  Arkani-Hamed/Motl/Nikolis/Vafa '06

  $$m < gM_P \qquad \text{or} \qquad \Lambda < gM_P \ ,$$

  has recently been revisited by many authors:

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14

  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler; Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski; Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala; Harlow; AH/Rompineve/Westphal; . . . '15

  Conlon/Krippendorf; Ooguri/Vafa; Freivogel/Kleban; Banks; Danielsson/Dibitetto; . . . . . . '16

Motivation (continued)

- An important new motivation is the axionic case,

$$g \equiv 1/f \ ,$$

relevant for natural inflation.

- Also, the domain wall case, is relevant for monodromy models.

- However, the basic origin of a possible Weak Gravity Theorem remains obscure.

see however Cottrell/Shiu/Soler '16

- Progress towards establishing such a theorem is important, even if (at first) only in 'exotic' cases.

Weak Gravity Conjecture for 2-forms

- We will study the dual side of the 'natural inflation case':

$$\int \frac{1}{f^2}\,|dB_2|^2 \;+\; \int_{\text{string}} B_2 \qquad \text{for} \qquad f \ll M_P\,.$$

- Formally extending the WGC to this case implies

  (a) Electric: Light strings with tension $\sigma < f\, M_P$

  or

  (b) Magnetic: A cutoff $\Lambda < \sqrt{f\, M_P}$.

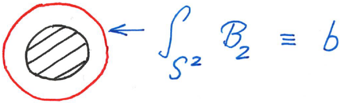
- The 'derivations' of both bounds are problematic since we would need 'extremal black strings' or 'black-hole instantons'.

Let us instead consider

Axionic Black Holes

Bowick/Giddings/Harvey/Horowitz/Strominger '88

- In the simplest case, these are just Schwarzschild BHs with a non-zero 'B_2-Wilson-line':

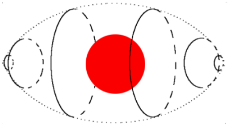


- Since the BH effectively induces a non-zero 2-cycle of space-time, such a non-zero $\langle b \rangle$ can be added at no cost to a standard BH solution.

Axionic Black Holes (continued)

- The non-zero 'Wilson-line' b can in principle be measured by strings 'lassoing' the BH.

Illustration from recent paper by Dvali/Gußmann:



- There is some controversy concerning the observability of this effect, but we believe this does not affect our parameter ranges.

Preskill/Krauss '90; Coleman/Preskill/Wilczek '92

Axionic Black Hole evaporation – explosive

- Now let the BH Hawking-radiate, as usual.

- R goes down, T goes up,
  nothing unusual happens before they reach

$$R_c \equiv 1/\sqrt{\sigma} \qquad \text{and} \qquad T_c \equiv \sqrt{\sigma}$$

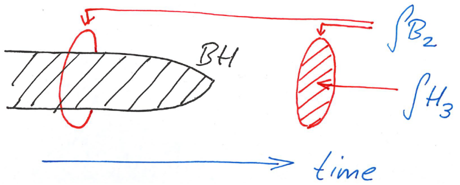
  or, alternatively,

$$R_c \equiv 1/\Lambda \qquad \text{and} \qquad T_c \equiv \Lambda\,.$$

- Let us assume that, at this moment, the BHs life ends on a short time scale $\sim R_c$
  (e.g. due to a KK or string tower-of-states).

Axionic Black Hole evaporation – explosive (continued)

- With the BH gone, the non-zero B_2 integral **must** be supported by field-strength (flux) of $H_3 = dB_2$



- Using $b = \oint B_2 = \int H_3$, we can estimate the energy of the resulting field configuration as

$$E \sim \frac{1}{f^2} \int |H_3|^2 \sim \frac{b^2}{f^2 R_c^3} \sim \frac{1}{f^2 R_c^3} .$$

Axionic Black Hole evaporation – explosive (continued)

- The necessary condition $E < M(R_c) \sim R_c M_P^2$ then immediately gives

$$\frac{1}{f^2 R_c^3} < R_c M_P^2 \qquad \text{and hence} \qquad \frac{1}{R_c^4} < f^2 M_P^2 \,.$$

- Recalling that $R_c = 1/\sqrt{\sigma}$, we now have

$$\sigma < f\, M_P \qquad \text{or} \qquad \Lambda^2 < f\, M_P \,,$$

i.e. precisely what is expected from the WGC.

Axionic Black Hole evaporation – slow

- Next, let us assume that nothing dramatic happens when the BH reaches

$$R_c \equiv 1/\sqrt{\sigma} \qquad \text{and} \qquad T_c \equiv \sqrt{\sigma}\,.$$

- However, unavoidably, virtual strings will start lassoing the BH and hence the variable

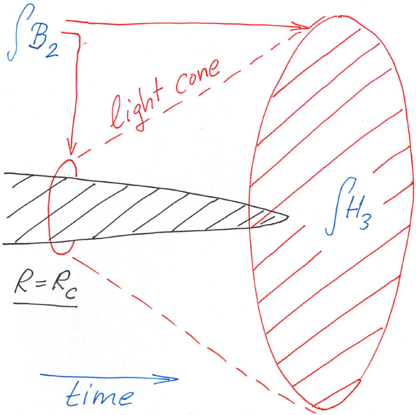
$$b(r) \equiv \int_{S^2(r)} B_2(r,\theta,\varphi)$$

  will start experiencing an effective force at $r \sim R_c$.

- $b(r)$ will develop a non-trival profile in r,
  and $H_3 = dB_2$ will have time to spread until the BH is gone.

Axionic Black Hole evaporation – slow (continued)

- Crucially, the resulting H_3 can be much more dilute than in the ‘explosive’ case:



Axionic Black Hole evaporation – slow (continued)

- The evaporation time from critical radius to 'zero' is

$$t_{ev} \sim \frac{M_c^3}{M_P^4} \sim R_c^3 M_P^2 \sim \frac{M_P^2}{\sigma^{3/2}} \,.$$

- Then H_3 can maximally spread to a radius $\sim t_{ev}$.

- Demanding that the corresponding energy satisfies $E < M(R_c)$, we now find

$$\sigma \sim \Lambda^2 \lesssim f^{2/5} \cdot M_P^{8/5} \,.$$

- This is much weaker than the naive WGC bound $\sigma < f \cdot M_P$.

- We expect a more careful analysis to give a bound in between our 'explosive' and 'slow' limits.

What if the WGC is violated only in the effective theory?

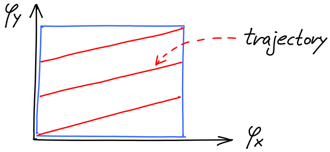
- As is well-known, with appropriate instanton choice, an axion with large f_{eff} can in principle follow from two small-f axions.

  Kim/Nilles/Peloso '04 (Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14)

- The possibly simplest way to achive such an effective small coupling is via Kaloper-Sorbo gauging, as in 'winding inflation'

  AH/Mangat/Rompineve/Witkowski '14

$$|F_0|^2 \quad \to \quad |F_0 + \varphi_1 + N\varphi_2|^2$$



- This can of course be done more generally, trying to evade e.g. the WGC for 1-forms in the effective theory.

  Saraswat '16

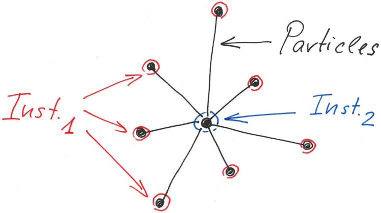
What if the WGC is violated only in the effective theory? (continued)

- We start with two 2-forms, $H_i = dB_i$, with

$$\mathcal{L} \sim \frac{1}{f^2}(|H_1|^2 + |H_2|^2) + \frac{1}{g^2}|dA + B_1 + NB_2|^2 \,.$$

- A single 2-form with $\sim (NB_1 - B_2)$ with $f_{eff} \sim f/N \ll 1$ will survive at low energies.

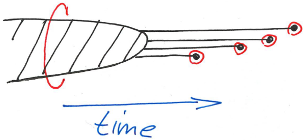
- It couples to a composite effective instanton, built from particles (monopoles) and the two fundamental instantons:



in analogy to Saraswat '16
(see also AH/Henkenjohann/Witkowski '17)

What if the WGC is violated only in the effective theory? (continued)

- We see that 'morally' $F = dA$ is identified with B, such that particles carry axionic charge.
- The axionic BH can now end by emitting some of those particles (ending on 'fractional' instantons)



- However, this does not provide us with a free lunch: We generically need $\mathcal{O}(N)$ of those, with $N = f/f_{eff}$.
- For $f = \mathcal{O}(M_P)$, we recover the WGC-scale Λ – no gain...

Two comments:

Quantum vs. classical

- Our 'axionic charge' b is actually a
  a periodic quantum mechanical variable
  (like a Wilson-line in a compactification to $d = 1$).

- We checked that our classical treatment is justified since the time scales involved are below the 'quantum-spread-time'.

Scalar perspective

- It is interesting to understand what the dual (scalar-field) picture for $b = \int_{S^2} B_2$ is.

- One needs to think about superimposing $\partial\varphi$-fluxes on the dual 1-cycle (in our case the radial direction).

- It turns out that b is the phase of this superposition
  (just like the familiar θ-angle).

Summary/Conclusions

- Barring various simplifying assumptions
  (which may or may not be innocent)
  we gave a new argument for a WGC-like-bound for 2-forms.

- Violation leads either to very exotic remnants, or to rather serious dynamical problems (stronger than the stable-BH or BH-monopoles issue?).

- Clearly, a much more careful analysis of the final moments of an axionic BH is desired.

- Most importantly, generalizations of the underlying idea to other dimensions/p-forms are needed.

  (An idea: We need to make sure that topology change through shriking cycles is dynamically consistent.)