Hence in 5d fermions are described by 4-component spinors. But now γ_5 is part of the algebra!

→ There are no chiral fermions like we had in 4d.
(Orbifold boundary conditions can also resolve this, see below.)

The 5d Lorentz invariants one can construct out of two four-component 5d spinors ψ_1 and ψ_2 are

$\bar{\psi}_1 \psi_2$ (like 4d Dirac mass term)

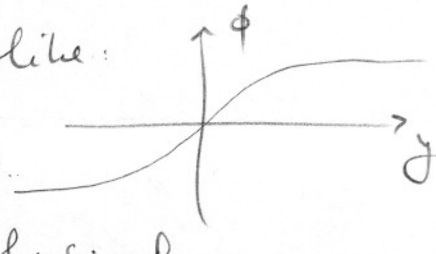
$\psi_1^T C_5 \psi_2$ (like 4d Majorana mass term)

where

$C_5 = \gamma^0 \gamma^2 \gamma^5$ is 5d charge conjugation

Now introduce a scalar field ϕ that forms a domain wall, like:

(note sign change at $y=0$)



This will lead to a physical example of a "fat brane", a brane of finite width.

Consider fermion in background $\phi(y)$ and assume Yukawa coupling to ϕ

$\rightarrow \quad S = \int d^4x \, dy \; \bar{\psi} \left[i\gamma_\mu \partial^\mu + i\gamma_5 \partial_y + \phi(y) \right] \psi ,$

leading to 5d Dirac equation

$$\left[i\gamma_\mu \partial^\mu + i\gamma_5 \partial_y + \phi(y) \right] \psi = 0$$

We look for solutions which are 4d left- or righthanded modes, hence we expand

$$\psi(x,y) = \sum_n \langle y | L, n \rangle \, P_L \, \psi_n^{(4)}(x) + \sum_n \langle y | R, n \rangle \, P_R \, \psi_n^{(4)}(x)$$

with

$$P_{L,R} = \frac{1}{2} \left(1 \pm \gamma_5 \right)$$

To do so we diagonalize the y-dependent part of Dirac operator, hence we have to find solutions to (as usual squaring the operator)

$$\underbrace{\left[-\partial_y^2 + \phi(y)^2 \pm \dot{\phi}(y) \right]}_{\substack{= a a^+ \\ (a^+ a)}} \left| \begin{smallmatrix} L \\ R \end{smallmatrix}, n \right\rangle = \mu_n^2 \left| \begin{smallmatrix} L \\ R \end{smallmatrix}, n \right\rangle$$

$$\dot{\phi} = \frac{d}{dy} \phi$$

for $a = \partial_y + \phi(y) \; , \quad a^+ = -\partial_y + \phi(y)$

With a, a^+ the problem can be formulated as a typical SUSY quantum mechanics problem:

$$Q = a \gamma^0 P_L$$
$$Q^+ = a^+ \gamma^0 P_R$$

gives for the Hamiltonian

$$H = \{Q, Q^+\} .$$

For such problems it is known that the eigenvalues (here for L and R modes) always come in pairs, <u>except</u> possibly for zero modes.

→ zero modes could give chiral 4d fermions.

Therefore we consider only zero modes, $n=0$, they have $\mu_n^2 = 0$.

$H\psi = 0$ is solved by $Q\psi = 0$, $Q^+\psi = 0$

$$\rightarrow \quad [\pm \partial_y + \phi(y)] \, |{}^{L}_{R}, 0\rangle = 0 ,$$

with solutions

$$\langle y | L, 0 \rangle \sim \exp\left[- \int_0^y \phi(s)\, ds \right]$$
$$\langle y | R, 0 \rangle \sim \exp\left[\int_0^y \phi(s)\, ds \right]$$

In infinitely large extra dim. they cannot be both normalizable.

→ Only one of them is relevant and we obtain a chiral zero-mode.

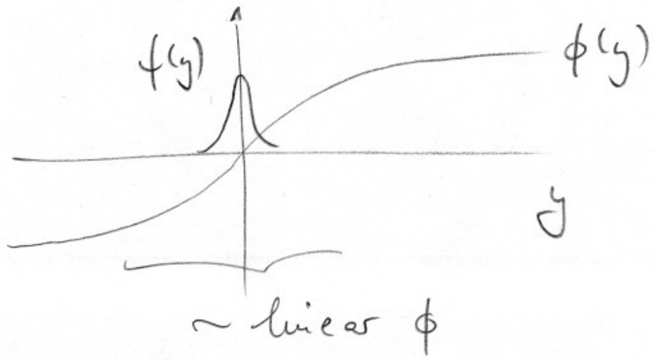
[In "realistic" models with a finite extra dimension the other mode can enter as well, but would be localized "at the other end of the extra dim."]

Assuming for example a linear domain wall (linear at least in a sufficiently large region around $y=0$)

$$\phi(y) \sim 2\mu^2 y$$

we have

$$\langle y | L, 0 \rangle = \frac{\sqrt{\mu}}{\left(\frac{\pi}{2}\right)^{1/4}} e^{-\mu^2 y^2}$$



while $\langle y | R, 0 \rangle$ is not normalizable and decouples.

Hence we obtain one left-handed fermion plus an infinite tower of massive Dirac fermions.

The zero-mode has a Gaussian wave function centered around $y=0$, that is where $\phi(y)$ changes sign. $\rightarrow$ The chiral fermion is <u>dynamically</u> localized to the domain wall. (often the name "domain wall fermion" is used, they are extensively used in lattice gauge theory.)

Next we put <u>several</u> fermions into the bulk. Then the action is

$$S = \int d^4x\, dy\, \bar{\Psi}_i \left[i\gamma_\mu \partial^\mu + i\gamma_5 \partial_y + \lambda\phi(y) - m \right]_{ij} \Psi_j$$

with a general mass matrix m_{ij} and a matrix λ_{ij} of Yukawa couplings.
For simplicity we assume $\lambda_{ij} = \delta_{ij}$ so that we can diagonalize m_{ij} with eigenvalues m_i.
We can find solutions in an analogous way, but now chiral fermions are localized around the zeros of

$$\phi - m_i .$$

If the domain wall is linear in a sufficiently large region, this gives fermions localized at

$$y^i = \frac{m_i}{2\mu^2},$$

that is, the m_i fix the relative position of the fermions along y.

Let us now turn to couplings in such models. For the SM we need fields of type:

doublet lefthanded
singlet righthanded,

however, only left handed fields emerge.
→ take charge conjugation for right-handed fields
Hence consider doublet L, singlet $\bar{E}^c$.
Consider Yukawa couplings to bulk Higgs field connecting fermions:
5d action is

$$S = \int d^5x \; \bar{L} \left(i\gamma^M \partial_M + \phi(y) \right) L$$
$$+ \int d^5x \; \bar{E}^c \left[i\gamma^M \partial_M + \phi(y) - m \right] E^c$$
$$+ \int d^5x \; \kappa \; H L^T C_5 E^c$$

As before we find that the lefthanded fermion l from L is localized at $y=0$, while the righthanded e^c from E^c is localized at $y = r = \frac{m}{2\mu^2}$.

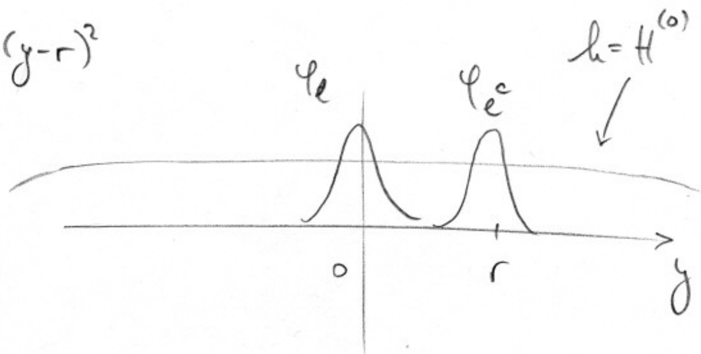
Then the effective 4d Yukawa coupling for zero modes will be

$$\int d^4x \, \kappa H \, l e^c \int dy \, \varphi_l(y) \, \varphi_{e^c}(y)$$

zero mode wavefunctions

but

$$\int dy \, \varphi_l(y) \, \varphi_{e^c}(y)$$
$$= \frac{\sqrt{2}\mu}{\sqrt{\pi}} \int dy \, e^{-\mu^2 y^2} e^{-\mu^2 (y-r)^2}$$
$$= e^{-\mu^2 r^2/2}$$



→ effective 4d Yukawa coupling is

$$\kappa \, e^{-\mu^2 r^2/2}$$

which is exponentially small even for bulk Yukawa coupling of order $O(1)$.

→ Split fermions can generate a large fermion hierarchy by relatively small splittings in the extra dimensions.

In this scenario fermion masses are thus related to geography in the extra dimension!

Similarly, one can suppress proton decay by localizing quarks and leptons at different ends of the fat brane. A dangerous operator would be

$$\int dy \, \frac{1}{M_*^3} \, (Q^T C_5 L)(U^{c\,T} C_5 D^c)$$

(as can be generated by quantum gravity). With overlap of wave functions one get an effective 4d coupling suppressed by

$$\int dy \, \left(e^{-\mu^2 y^2}\right)^3 e^{-\mu^2 (y-r)^2} \sim e^{-\frac{3}{4}\mu^2 r^2} .$$

→ An arbitrarily large suppression is possible without invoking any symmetry.

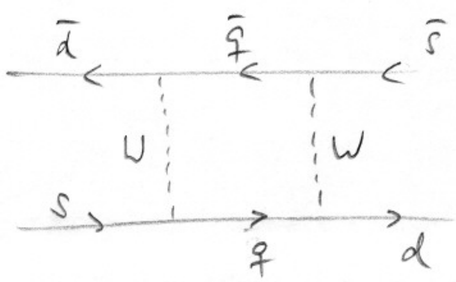
One can in fact construct large ($\sim \mathrm{TeV}^{-1} \cdot 10^{\frac{32}{n}}$) extra dimension scenarios with a fat brane that are "realistic": gauge and Higgs fields propagate throughout the fat brane, and only gravity propagates in full 5d spacetime. For a width of the fat brane $\sim \mathrm{TeV}^{-1}$ the KK modes of the gauge fields are sufficiently heavy. The typical width of the fermions in y is $\sim 0.1 - 0.01 \ \mathrm{TeV}^{-1}$.

2) Mediation of SUSY breaking via extra dimensions

Let us assume that the SM hierarchy problem is resolved by supersymmetry. Then the important question is: how is SUSY broken? Usually, in the MSSM (minimal supersymmetric SM) soft SUSY-breaking parameters are put in by hand, but they essentially only parametrize our ignorance of the real mechanism.

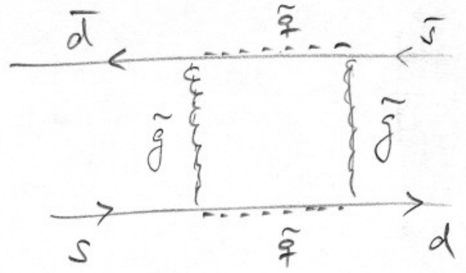
A very strong constraint on such parameters comes from the absence of flavor-changing neutral currents.

To illustrate the problem, consider K-$\bar{K}$ mixing which is very small in the SM due to the GIM mechanism:



This is prop. to off-diagonal elements of $V^{+}_{CKM} V_{CKM}$
→ zero due to unitarity of V_{CKM}.

In the MSSM a possible contribution is



with squarks and gluinos, prop. to

$V^{+}_{CKM} \, M^{2}_{squarks} \, V_{CKM}$

This is small only if $M_{squark} \sim$ unit matrix.

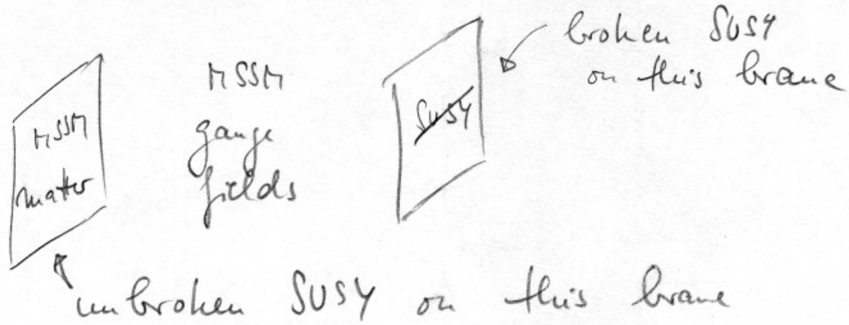
→ "SUSY flavor problem": Why should squarks be degenerate in mass?

In extra dimensions, there are new ways of attacking the SUSY breaking problem.
Two important proposals are

- anomaly mediation
- gaugino mediation

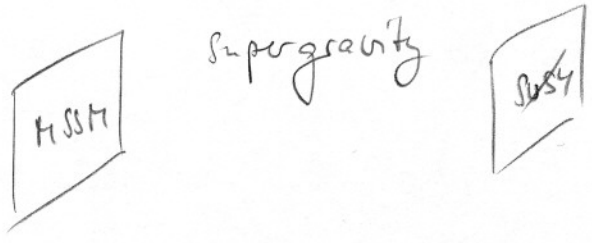
In both cases the assumption is that there are two branes separated by some (not necessarily large!) distance in an extra dimension. On one brane ("our world") SUSY is unbroken while on the other brane SUSY is explicitly broken.

In the case of gaugino mediated SUSY breaking one lets propagate the MSSM gauge fields in the bulk,



Then gauginos couple directly to SUSY breaking on the other brane, hence the leading breaking effect will be in the gaugino sector, while scalars experience SUSY breaking only via loop effects (with gauge fields propagating to the other brane) such that SUSY breaking effects are smaller in that sector.

In <u>anomaly mediated</u> SUSY breaking one has two similar branes but now lets only supergravity fields propagate in the bulk.



Now <u>all</u> SUSY-breaking effects in the MSSM are through loop effects. (Hence the name: this is a pure quantum effect, like anomalies in gauge theories)

3) Universal extra dimensions (UED)

(Appelquist, Cheng, Dobrescu)

In this model all SM fields propagate in all dimensions (hence the name "universal"). One assumes flat extra dimensions with compactification radius $R \sim \mathrm{TeV}^{-1}$ (therefore one also speaks of "TeV-sized extra dim."), that is much smaller than in the ADD model. Essentially, the idea of this model is closer to the original KK model than to ADD.

Models of this kind have an interesting phenomenology, in particular they provide good candidates for dark matter, namely the lightest KK-state, which can be stable.

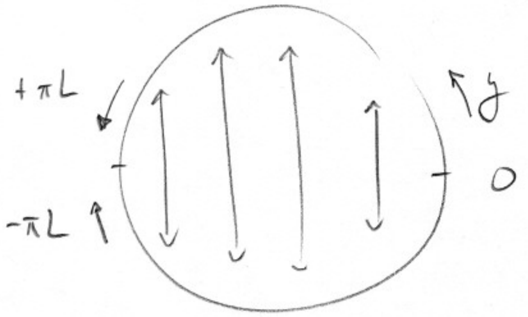
UED models are also often used as prototype models for effects of extra dimensions at colliders.

In these lectures, however, we will not consider any details of UED models.

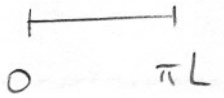
4) Orbifold compactification

As a motivation recall the chirality problem (no chiral representation of the Clifford algebra in 5d!). One way to resolve this is orbifold compactification, but orbifolds have also other effects that are interesting for model building.

An orbifold is in the simplest case obtained by compactifying one dimension on a circle S^1 with periodic boundary conditions and identifying y with $-y$:



The resulting physical space is an interval



with certain boundary conditions,

$\rightarrow$ the orbifold $S^1/\mathbb{Z}_2$, since $y \leftrightarrow -y$

corresponds to a (discrete) $\mathbb{Z}_2$-transformation.

To implement the identification $y \leftrightarrow -y$ on the fields, assign a parity P under the Z_2-transformation to all fields (to be respected by the action).

For example:

$$\left.\begin{aligned} P A_\mu &= + A_\mu \\ P \psi_L &= + \psi_L \\ P \psi_R &= - \psi_R \end{aligned}\right\}$$ assuming vanishing 5d fermion mass

Then all fields have

$$\phi(x^\mu, -y) = P(\phi)\,(x^\mu, y),$$

and we also have periodicity

$$\phi(x^\mu, y + 2\pi L) = \phi(x^\mu, y).$$

Together these specify "orbifold boundary conditions" on the interval.

To satisfy these boundary conditions:

$$A_\mu^{(n)},\ \psi_L^{(n)} \sim \cos\frac{ny}{L}$$

$$\psi_R^{(n)} \sim \sin\frac{ny}{L}$$

Obviously, for $n=0$ $\psi_R^{(0)} = 0$, and there is no right-handed fermion zero-mode! Only a massless left-handed fermion is left.

→ Orbifold compactification can resolve the chirality problem.

Orbifolds can also be used in many other ways to project out unwanted states.

→ many applications of orbifolds in extra-dimensional model building.

5. Warped extra dimensions (Randall - Sundrum models)

So far we have considered flat extra dimensions and have neglected the brane tension.

→ What is actually the interplay of brane tensions and bulk cosmological constant?

As we will see, the mutual back-reaction of branes and bulk can be quite relevant: the effect of brane tension can be balanced by a bulk cosmological constant.

→ curved space with flat brane(s) becomes possible!

1) RS 1 model

In the RS 1 (Randall-Sundrum-1) model consider one extra dimension with orbifold compactification S^1/Z_2 and relevant interval $[0, L]$

→ Space is $\underbrace{\mathbb{R}^4}_{\text{4d Minkowski}} \times S^1/Z_2$

The bulk action is

$$S_{bulk} = \int d^4x \int_{-L}^{L} dy \sqrt{|G|} \left(2M_*^3 R - \Lambda \right)$$

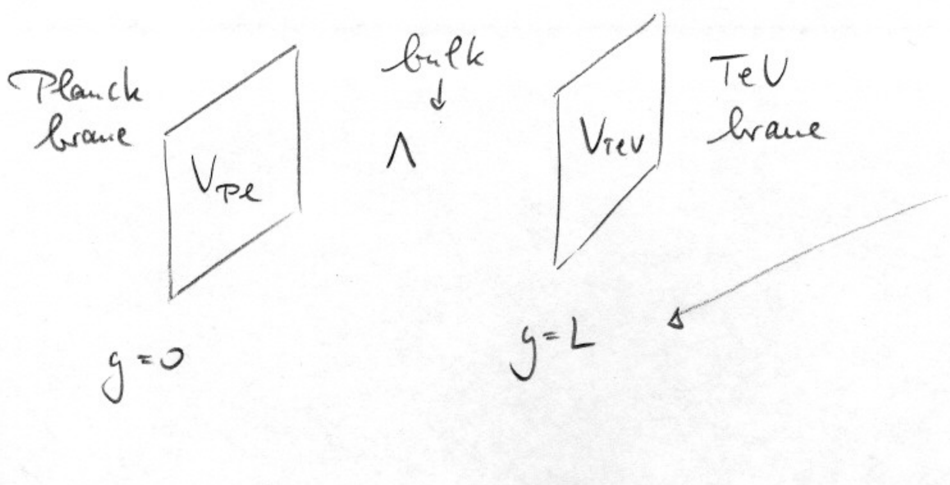
$$= 2 \int d^4x \int_{0}^{L} dy \sqrt{|G|} \left(2M_*^3 R - \Lambda \right)$$

to see how orbifold boundary conditions act it is convenient to study two copies glued together back-to-back

Then put two branes – called <u>Planck-brane</u> and <u>TeV-brane</u> – at $y=0$ and L that have tensions V_{PE}, V_{TeV} (4d cosmological constants) and matter fields described by $\mathcal{L}_{PE}$ and $\mathcal{L}_{TeV}$. Both couple to the 4d components of the bulk metric, that is to <u>induced metric</u>

$$g_{\mu\nu}^{PE}(x^\mu) = G_{\mu\nu}(x^\mu, y=0)$$

$$g_{\mu\nu}^{TeV}(x^\mu) = G_{\mu\nu}(x^\mu, y=L)$$



Note:
L is <u>not</u> necessarily large

Hence the brane actions

$$S_{Pl} = \int d^4x \sqrt{|g_{Pl}|} \left(\mathcal{L}_{Pl} - V_{Pl} \right)$$

$$S_{TeV} = \int d^4x \sqrt{|g_{TeV}|} \left(\mathcal{L}_{TeV} - V_{TeV} \right)$$

→ total action:

$$S = S_{bulk} + S_{Pl} + S_{TeV}$$

Now consider only effect of gravity and tension / cosmological constant, and neglect matter on the branes.

→ 5d Einstein equations for this:

$$\sqrt{|G|} \left(R_{MN} - \frac{1}{2} G_{MN} R \right) =$$

$$= - \frac{1}{4 M_*^3} \Big[\Lambda \sqrt{|G|} \, G_{MN} + V_{TeV} \sqrt{|g_{TeV}|} \, g^{TeV}_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(y-L) + V_{Pl} \sqrt{|g_{Pl}|} \, g^{Pl}_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(y) \Big]$$

We look for a solution that respects 4d Poincaré invariance in x^μ-directions

→ non-factorizable ansatz for 5d metric:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

With this ansatz Einstein eq. gives

$$6\sigma'^2 = -\frac{\Lambda}{4M_*^3} \qquad (*)$$

$$3\sigma'' = \frac{1}{4M_*^3} V_{Pl}\,\delta(y) + \frac{1}{4M_*^3} V_{TeV}\,\delta(y-L) \qquad (**)$$

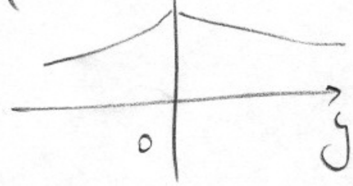
For orbifold we require symmetry w.r.t. $y \leftrightarrow -y$, so from $(*)$:

$$\sigma = |y| \sqrt{\frac{-\Lambda}{24 M_*^3}}$$

and a reasonable (real-valued) solution requires $\Lambda < 0$, that is a constant <u>negative curvature</u>. Such a space is called an <u>Anti-de Sitter space</u>, in 5d also denoted by <u>AdS_5</u>.

More precisely, since we are dealing with a space bounded by two branes, a <u>slice of AdS_5</u>.

For second derivative of σ recall orbifold boundary conditions
→ a cusp is to be expected and hence the δ-fct. pieces in eq. $(**)$ for σ''.



With the solution to (*) we hence have to fulfill in addition for $-L \leq y \leq L$:

$$\sigma'' = 2\sqrt{\frac{-\Lambda}{24 M_*^3}} \left[\delta(y) - \delta(y-L) \right] .$$

A solution is obtained for (**) only if V_{TeV}, V_{Pl} and Λ are related. Then they can be expressed in terms of a single scale k:

$$V_{Pl} = - V_{TeV} = 24 M_*^3 k$$
$$\Lambda = - 24 M_*^3 k$$

Obviously, a fine-tuning is required to have a solution.

Note also that L is not determined and remains as a free parameter — a "modulus". Changes of L correspond to a massless field, the radion.

To make the model realistic, some stabilization mechanism is required (see below).

In summary, we have

$$\sigma(y) = k|y|$$

with

$$k = \sqrt{\frac{-\Lambda}{24 M_*^3}}$$

k is called the <u>AdS curvature</u>, while $l = \frac{1}{k}$ is the <u>AdS length</u>.

The bulk metric is

$$ds^2 = \underbrace{e^{-2k|y|}}_{\text{warp factor}} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

Recall that the TeV-brane has negative tension ($\rightarrow$ "negative tension brane" or "weak brane") and the Planck-brane has positive tension ($\rightarrow$ "positive tension brane", "gravity brane") in order to have a solution to the Einstein equation.

2) Physical scales in RS1

Now consider physical mass scales in this model.
We assume that matter resides on the TeV-brane, for example the Higgs field.
The action for the latter is

$$S^{Higgs} = \int d^4x \sqrt{|g^{TeV}|} \left[g_{\mu\nu}^{TeV} D^\mu H D^\nu H - V(H) \right]$$

with the Higgs potential

$$V(H) = \lambda \left(H^+ H - v^2 \right)^2 .$$

With the warp factor the induced metric is

$$g_{\mu\nu}^{TeV} = e^{-2kL} \eta_{\mu\nu} .$$

Inserting into the action:

$$S^{Higgs} = \int d^4x \; e^{-4kL} \left[e^{2kL} \eta_{\mu\nu} D^\mu H D^\nu H + \lambda \left(H^+ H - v^2 \right)^2 \right]$$

To obtain canonically normalized Higgs field we re-define $\hat{H} = e^{-kL} H$, so

$$S^{Higgs} = \int d^4x \left\{ \eta_{\mu\nu} D^\mu \hat{H} D^\nu \hat{H} - \lambda \left[\hat{H}^+ \hat{H} - \left(e^{-kL} v \right)^2 \right]^2 \right\}$$

That is the standard form of the Higgs action, but with a VEV that is warped down to

$$\tilde{v}_{Higgs} = e^{-kL} v .$$

Note that this scale (which sets all SM mass parameters!) is now <u>exponentially smaller</u> than usually.

One finds that this mechanism is completely general: <u>all</u> masses are exponentially suppressed on the TeV-brane. But this is not the case on the Planck-brane (positive tension brane) since there the warp factor equals 1.

That phenomenon is sometimes called "red shifting" of all energy scales away from the positive tension brane.

Interpretation: all fundamental mass parameters are of $\mathcal{O}(M_{Pl})$, including $v \sim 0.1\, M_{Pl}$ on the Planck brane, and are warped down exponentially on the TeV brane.

Then to obtain

$$\tilde{v} \sim 0.1 \cdot e^{-kL} M_{Pl} \sim 0.1 \text{ TeV}$$

requires only

$$k \cdot L \sim 35 .$$

That is a rather simple physical requirement without too much fine-tuning. Then all dimensionful parameters on the TeV-brane are warped down, and for that one only needs that the size of the AdS space L is parametrically larger than the inverse curvature.

Hence the actual parameter of the extra dimension in RS 1 is <u>not</u> the size L, but rather the (inverse) curvature of the AdS space.

[Another possible interpretation is that all fundamental parameters are of $\mathcal{O}$(TeV) and M_{Pl} is large due to "warping up" in the opposite direction.]

What is 4d effective scale of gravity (4d Planck scale)?

$\to$ find coefficient of $R^{(4)}$ upon integrating out y-direction.

The 4d graviton is embedded as (see later)

$$ds^2 = e^{-2k|y|}\left[\eta_{\mu\nu} + h_{\mu\nu}(x)\right]dx^\mu dx^\nu - dy^2$$

Then $R^{(5)}_{\mu\nu}$ contains $R^{(4)}_{\mu\nu}$ calculated from $h_{\mu\nu}$ (since $R_{\mu\nu}$ is invariant under rescaling of the metric)

$$\to \quad S = -M_*^3 \int d^5x \sqrt{G}\, R^{(5)}$$

contains

$$-M_*^3 \int d^5x \, e^{-4k|y|} \sqrt{g^{(4)}}\, e^{2k|y|}\, R^{(4)}$$

Hence

$$M_{Pl}^2 = M_* \int_{-L}^{L} e^{-2k|y|}\, dy$$

$$= \frac{M_*^3}{k}\left(1 - \underbrace{e^{-2kL}}_{\text{small}}\right)$$

[note: this result is finite for $L \to \infty$!?]

Then it is natural that

$$M_{Pl} \sim k \sim M_*$$

are all of Planck scale size.