Further, we can fix gauge corresponding to D-dim. general coord. invariance by harmonic gauge (for example)

$$\partial_M h^M_{\ N} = \frac{1}{2}\partial_N h^M_{\ M}$$

This is not yet complete gauge fixing, still gauge transformation

$$h_{MN} \to h_{MN} + \partial_M \epsilon_N + \partial_N \epsilon_M$$

with

$$\Box \epsilon_M = 0$$

left, hence another D conditions can be imposed. After that

$$\frac{1}{2} D(D+1) - 2D = \frac{1}{2} D(D-3)$$

degrees of freedom left.

In $D=4$ that are 2 helicity states, $D=5$ gives 5 components, $D=6$ gives 9 etc.

→ higher-dim. graviton involves more fields!

Consider in the following the different components of graviton

⊕ 4D <u>graviton and KK modes</u>:

$$4\left\{\left(\begin{array}{c|c} \overbrace{\quad G^{\vec{k}}_{\mu\nu} \quad}^{4} & \\ \hline & \end{array}\right)\right. \qquad \vec{k} \text{ is } n\text{-comp. vector,}$$

For no sources they satisfy

$$(\Box + \hat{k}^2)\, G^{\vec{k}}_{\mu\nu} = 0$$

with

$$\hat{k}^2 = \sum_{i=1}^{n} \left|\frac{k_i}{R}\right|^2 .$$

Above eq. implies 10 components, but 5 eliminated by gauge conditions,

$$\partial^\mu G^{\vec{k}}_{\mu\nu} = 0 \quad , \quad G^{\mu\vec{k}}_{\mu} = 0$$

→ massive graviton in 4D : 5 d.o.f (eats one massless vector + massless scalar)

⊕ 4D vector and KK modes

$$\left(\begin{array}{c|c} & V^{\vec{k}}_{\mu j} \\ \hline V^{\vec{k}}_{j\mu} & \end{array}\right)$$

$n-1$ such KK towers, since graviton has eaten one. This's expressed in constraint

$$\hat{k}^j V^{\vec{k}}_{\mu j} = 0 .$$

In addition, usual Lorentz gauge

$$\partial^\mu V^{\vec{k}}_{\mu j} = 0 .$$

Massive vectors have eaten a scalar.

⊕ 4D scalars and KK modes

$$\left(\begin{array}{c|c} & \\ \hline & S^{\vec{k}}_{ij} \end{array}\right)$$

Originally, $\frac{1}{2} n(n+1)$. But one eaten by graviton, and $(n-1)$ eaten by vectors. One scalar mode is special since its zero-mode sets the size of the internal manifold. This special scalar is called <u>radion</u>.

Hence $\frac{1}{2} n(n+1) - n - 1 = \frac{1}{2}(n^2 - n - 2)$ scalars left.

n eaten fields correspond to constraint

$$\hat{k}^j S^{\vec{k}}_{jk} = 0,$$

separating out radion by constraint

$$S^{\vec{k}j}{}_{j} = 0.$$

Then radion given by $h^{\vec{k}j}{}_{j}$.

Total # of d.o.f.

$$5 \text{ (graviton)} + 3(n-1) \text{ (vectors)}$$
$$+ \frac{1}{2}(n^2-n-2) \text{ (scalars)} + 1 \text{ (radion)}$$
$$= \frac{1}{2}(4+n)(1+n) = \frac{1}{2} D(D-3)$$

as expected.

Explicitly (with usual normalization) in unitary gauge:

radion: $\quad H^{\vec{n}} = \frac{1}{k} h_j^{\vec{n}j}$

Scalars: $\quad S_{ij}^{\vec{n}} = h_{ij}^{\vec{n}} - \frac{k}{n-1}\left(\eta_{ij} + \frac{\hat{n}_i \hat{n}_j}{\hat{n}^2}\right) H^{\vec{n}}$

vectors: $\quad V_{\mu j}^{\vec{n}} = \frac{i}{\sqrt{2}} h_{\mu j}^{\vec{n}}$

gravitons: $\quad G_{\mu\nu}^{\vec{n}} = h_{\mu\nu}^{\vec{n}} + \frac{k}{3}\left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{\hat{n}^2}\right) H^{\vec{n}}$.

where $\quad \hat{n}_i = \frac{n_i}{R}, \quad k = \sqrt{\frac{3(n-1)}{n+2}}$.

In presence of sources $T^{\mu\nu}$ the equation of motion for above fields is

$$(\Box + \hat{n}^2)\begin{pmatrix} G_{\mu\nu}^{\vec{n}} \\ V_{\mu j}^{\vec{n}} \\ S_{ij}^{\vec{n}} \\ H^{\vec{n}} \end{pmatrix} = \begin{pmatrix} \frac{1}{M_{Pl}}\left[-T_{\mu\nu} + \frac{1}{3}\left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{\hat{n}^2}\right) T^\mu_\mu\right] \\ 0 \\ 0 \\ \frac{k}{3 M_{Pl}} \, T^\mu_\mu \end{pmatrix}$$

Hence radion is only field besides 4D graviton that couples to SM sources! Vector and other scalars are not important for SM matter - bulk graviton coupling.

As example, consider QED on the brane,

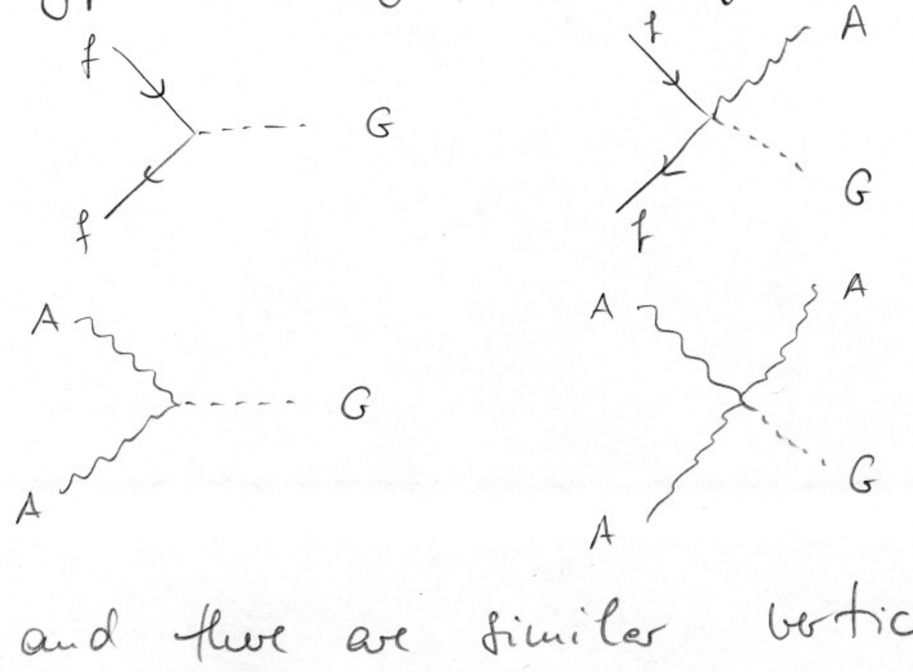
$$\mathcal{L} = \sqrt{|g|}\,\left(i\bar{\psi}\gamma^a D_a \psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \right)$$

then energy-momentum tensor is

$$T_{\mu\nu} = \frac{i}{4}\,\bar{\psi}\left(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu\right)\psi - \frac{i}{4}\left(\partial_\mu \bar{\psi}\gamma_\nu + \partial_\nu \bar{\psi}\gamma_\mu\right)\psi$$
$$+ \frac{1}{2}\, eQ\, \bar{\psi}\left(\gamma_\mu A_\nu + \gamma_\nu A_\mu\right)$$
$$+ F_{\mu\lambda}F_\nu{}^\lambda + \frac{1}{4}\eta_{\mu\nu} F^{\lambda\rho}F_{\lambda\rho} .$$

→ Find QED - graviton couplings and Feynman rules from linear coupling to $h_{\mu\nu}$; similarly for whole SM.

Typical Feynman diagrams are



← (non-Abelian gauge theory)

and there are similar vertices for the radion.

6) Phenomenology of large extra dimensions

Recall that mass splitting of KK modes is extremely small:

$$\Delta m \sim \frac{1}{r} = M_* \left(\frac{M_*}{M_{Pl}}\right)^{\frac{2}{n}} = \left(\frac{M_*}{\text{TeV}}\right)^{\frac{n+2}{2}} 10^{\frac{12n-31}{n}} \text{ eV}.$$

Hence a large number of KK modes is available for being produced in scattering processes at high energy. To deal with this large number we can turn the sum over KK modes into an integral (almost continuum due to small spacing). Let N be number of KK modes with extra-dim. momentum below k, then

$$dN = S_{n-1}\, k^{n-1}\, dk$$

with $S_n = (2\pi)^{n/2} \cdot \frac{1}{\Gamma(\frac{n}{2})}$ the surface of n-dim. sphere of unit radius.

Mass of a given KK mode is $m = \frac{|k|}{R}$, hence

$$dN = S_{n-1}\, m^{n-1} R^{n-1}\, dm\, R$$

$$= S_{n-1} \frac{M_{Pl}^2}{M_*^{n+2}}\, m^{n-1}\, dm$$

If $\frac{d\sigma_n}{dt}$ is differential cross section for production of an individual mode of mass m, then we get

$$\frac{d^2\sigma}{dt\,dm} = S_{n-1} \frac{M_{Pl}^2}{M_*^{n+2}} m^{n-1} \frac{d\sigma_n}{dt} .$$

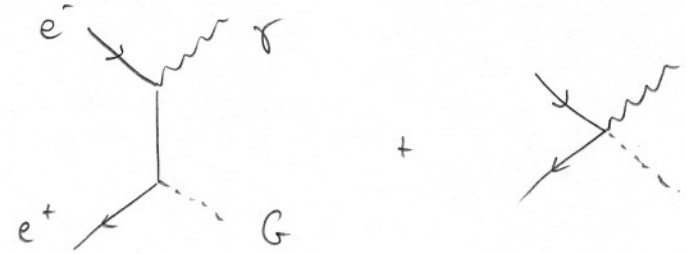
Recall that coupling of SM fields to individual KK mode is $\sim \frac{1}{M_{Pl}}$, hence corresponding cross section $\sim \frac{1}{M_{Pl}^2}$. Therefore, inclusive cross section will be

$$\frac{d^2\sigma}{dt\,dm} \sim S_{n-1} \frac{m^{n-1}}{M_*^{n+2}} .$$

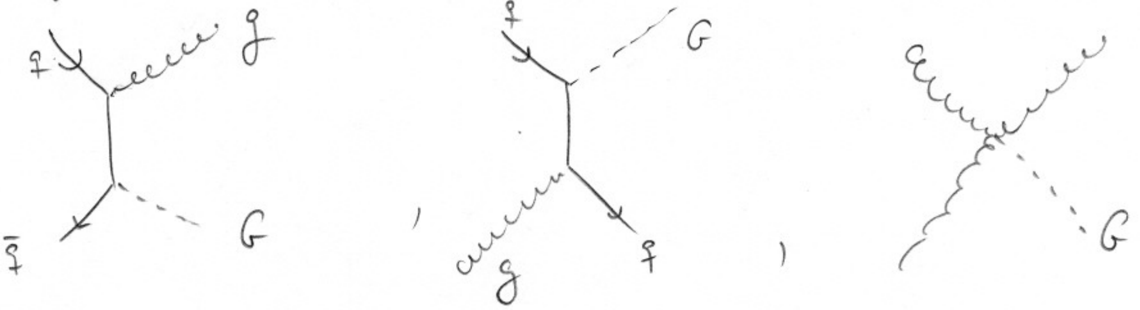
Here detailed phenomenology of many processes could start. We will only briefly discuss some interesting examples, concentrating on the results rather than the actual — sometimes lengthy — calculations.

* Graviton production at colliders

Typical process at e^+e^- - colliders is
$$e^+e^- \to \gamma G,$$
corresponding diagrams:



At hadron colliders one can have single - jet production from diagrams like



Individual graviton KK mode, once produced, has very long lifetime τ
$$\tau = \frac{1}{\Gamma} \sim \frac{M_{Pl}^2}{m^3} \quad \left(\text{since coupling to matter is} \sim \frac{1}{M_{Pl}}\right).$$

Therefore, graviton will not decay inside the detector, and will not be seen due to its weak interaction with matter.

$\to$ signature is missing energy!

In an e^+e^- collider a typical signal would be a single photon recoiling against missing energy, $e^+e^- \to \gamma + \not{E}_T$. At hadron colliders one would see a single jet and missing energy.

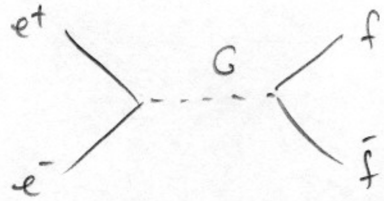
These processes have SM backgrounds (like Z production with initial state photon radiation and $Z \to$ neutrinos) which are rather small.

Importantly, such processes are often very different from SUSY processes.

As is clear from the general form of cross sections from higher-dim. perspective, such processes become relevant at energies $E \sim M_*$.

* virtual graviton exchange

At energies $E \geq M_*$ virtual graviton exchange becomes relevant:



with amplitude

$$A \sim \frac{1}{M_{pe}^2} \sum_{\vec{k}} \frac{1}{s - m_{kk}^2}$$

(Note: some problems due to UV divergence, $A \sim s^{(n-2)/2} \to$ UV completion?)

* Supernova cooling

Since graviton modes are weakly coupled they can carry away energy from supernovae. For temperature $T < r^{-1}$ no KK modes can be excited, but if $T \gg r^{-1}$ a large number $\sim (Tr)^n$ of modes can be produced. Each mode is coupled with strength $\frac{1}{M_{Pl}}$

$\rightarrow$ rate of graviton production is

$$\sim \frac{1}{M_{Pl}^2} (Tr)^n \sim \frac{T^n}{M_*^{n+2}}$$

with T typical temperature within a supernova ~ 30 MeV.

The normal cooling process is via neutrino emission. A considerable cooling via gravitons would hence affect the neutrino signal. No effect has been seen in SN 1987A, for example, where the signal was not shorter than expected.

This gives $M_* \gtrsim 30$ TeV for $n=2$.

* Supernova → neutron star transition

In a supernova many KK modes are produced near threshold, that is with low kinetic energy. Therefore they remain trapped in gravitational field of neutron star.

The decay time for $G \to \gamma\gamma$ is

$$\tau \sim 6 \cdot 10^9 \, yr \cdot \left(\frac{100 \, MeV}{m_G} \right)^3 .$$

Hence by now a significant fraction should have decayed into two hard photons, turning neutron stars into hard γ-sources. Non-observation of this implies

$$M_* \gtrsim 100 \, TeV \quad \text{for} \quad n = 2 .$$

* Cooling into the bulk

At high temperatures in the early universe a considerable fraction of the energy on the brane can be carried away (into the bulk) via graviton production.

→ Compare cooling due to graviton production with ordinary cooling due to Hubble expansion.

Via expansion, the energy density changes as

$$\left.\frac{d\rho}{dt}\right|_{\text{expansion}} \sim -3H\rho$$

$$\sim -3\frac{T}{M_{Pl}^2}\rho$$

while graviton emission gives

$$\left.\frac{d\rho}{dt}\right|_{\text{grav-emission}} \sim \frac{T^n}{M_*^{n+2}}\rho$$

The two rates are equal at the so-called normalcy temperature T_*, below which normal expansion would dominate. It is given by

$$T_* \sim \left(\frac{M_*^{n+2}}{M_{Pl}}\right)^{\frac{1}{n+1}} = 10^{\frac{6n-9}{n+1}}\ \text{MeV}$$

After inflation, the reheating temperature should hence stay below the normalcy temperature to avoid production

of too much dark matter (gravitons in bulk) leading to overclosure.
For $n=2$, $T_* \sim 10\,\mathrm{MeV}$, which is just barely sufficient for nucleosynthesis.
Generally, in these models baryogenesis is an enormously difficult problem due to the low temperature of the universe.

* Black hole production at colliders

Since the scale of quantum gravity is lowered to TeVs in the ADD model one can even expect black hole production at colliders.
Black holes are formed when the mass of an object is within the horizon corresponding to that mass.
The horizon for the mass of the Earth, for example, is $\sim 8\,\mathrm{mm}$.
Most of the mass of the Earth is clearly outside the horizon, and no black hole is formed.

To calculate typical size of horizon in large extra dimensions, consider first 4d case. Schwarzschild solution is given by

$$ds^2 = \left(1 - \frac{GM}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{GM}{r}\right)} + r^2 d^2\Omega$$

and the horizon is the distance at which the coefficient of dt^2 vanishes,

$$r_H^{(4)} = GM$$

In $4+n$ dimensions the prefactor of dt^2 is replaced by

$$1 - \frac{GM}{r} \longrightarrow 1 - \frac{M}{M_*^{2+n} r^{1+n}},$$

hence the horizon size

$$r_H^{(4+n)} \sim \left(\frac{M}{M_*}\right)^{\frac{1}{1+n}} \frac{1}{M_*}.$$

The exact solution gives similar result with some numerical coefficient.

In a particle collision, a black hole would form if the collision takes place with sufficient energy ($\rightarrow$ mass) at impact parameter below $r_H^{(4+n)}$.

Hence a black hole of mass $M_{BH} = \sqrt{s}$ forms roughly with geometrical cross section,

$$\sigma \sim \pi r_H^2$$

$$\sim \frac{1}{M_*^2} \left(\frac{M_{BH}}{M_*} \right)^{\frac{2}{n+1}}$$

giving about $\frac{1}{\text{TeV}^2} \sim 400$ pb.

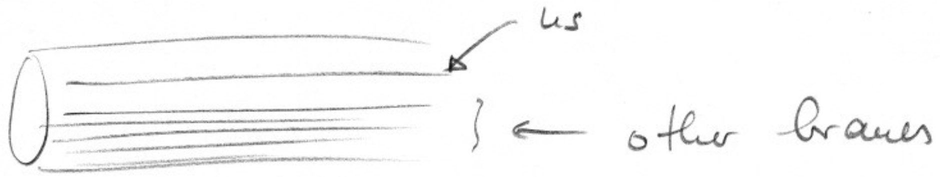
If so, the LHC would produce about 10^7 black holes per year!

They would be unstable and decay via Hawking radiation, in which all SM particles are produced with equal probability in a spherical distribution. A black hole of mass ~ 10 TeV would therefore decay into about ~ 50 particles with energies ~ 200 GeV each, leading to a spectacular signal.

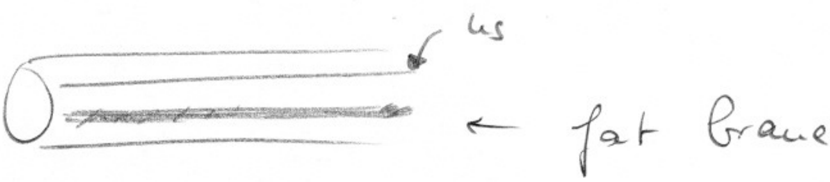
Various modifications of the ADD model have been proposed which can avoid some of the phenomenological bounds.

An example is to add further branes in the extra dimension.

- many other branes



- a "fat" brane



In such scenarios the gravitons would predominantly decay on the other branes, thus avoiding astrophysical bounds on production of hard photons via $G \to \gamma\gamma$.

4. Various models and mechanisms in flat extra dimensions

A number of different models with flat extra dimensions has been proposed, not all necessarily with large extra dimensions. Here we want to illustrate just a few interesting mechanisms.

1) Split fermions

If indeed extra dimensions are large and fundamental Planck scale is $M_* \sim 1\,\mathrm{TeV}$, then the following issue emerges:

Quantum gravity generically breaks all global symmetries (but not gauge symmetries). Hence global symmetries can only be assumed to hold up to operators suppressed by the scale of quantum gravity.

Baryon number is an accidental global symmetry of the SM (every renormalizable operator consistent with gauge symmetry also conserves baryon number, without explicitly requiring it). But one can write non-renormalizable operators that violate baryon number.

If in the SM there is no new physics up to the GUT or Planck scale then those operators have a $\frac{1}{M_{GUT/Pl}}$ - suppression, leaving the proton sufficiently long-lived. However, new physics beyond the SM could induce proton decay. For example in the MSSM one has to introduce R-parity to avoid proton decay, otherwise suppression would be only $\sim \frac{1}{M_{SUSY}}$.

In large extra dimensions the situation is worse: quantum gravity is expected to be sizeable at Planck scale $M_* \sim \mathrm{TeV}$. Thus proton decay via an operator with three quarks and one lepton, for example, would have a small suppression

$$\frac{1}{M_*^2} \, QQQL$$

and would cause proton decay.

A nice solution to this problem in extra dimensions was proposed by Arkani-Hamed and Schmaltz. They make use of the extra dimension to suppress the dangerous operators. The idea is to localize the SM fermions at slightly different points along the extra dimension

→ "<u>Split fermion scenario</u>"

If the fermions have narrow wave functions along the extra dim. the dangerous operators get suppressed by the small overlap of the corresponding wave functions. This idea can also be used to generate the fermion mass hierarchy.

So far we have not discussed the issue of localizing fields. The following discussion shows how a "brane-like" object can emerge in field theory as a domain wall.

We consider a single extra dimension and try to localize fermions. For that we first need to know how to describe fermions in 5d. A fermion is a representation of the Poincaré group, and we need a representation of the

5d Clifford algebra

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{\mu\nu}$$

to describe it. Such a representation is straightforward to find in terms of the well-known (from 4d) Dirac gamma-matrices:

$$\Gamma_\mu = \gamma_\mu^{(4d)} \quad , \quad \Gamma_5 = -i\gamma_5^{(4d)}$$

(In the following we will in 5d use again small γ's : $\gamma_M \equiv \Gamma_M$)

Note: in 5d this is the smallest representation of the Clifford algebra. Recall that in 4d in addition to the corresponding 4-component spinors one had also a 2dim representation given by Weyl fermions (chiral fermions)