GEOMETRY AND PHYSICS II
Institut Henri Poincaré, Nov. 28 &29, 2013

Modified Anti-de-Sitter Metric, Light-Front Quantized QCD, and Conformal Quantum Mechanics

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Modified Anti-de-Sitter Metric, Light-Front Quantized QCD, and Conformal Quantum Mechanics

1) Theory of Strong Interaction. The Problems

2) Modified Anti-de-Sitter metric. A solution?

3) AdS metric, LF Quantized QCD and Conformal Q M.

Une ménage à trois

Quantum Chromo Dynamics, theory of strongly interacting particles (hadrons: protons, neutrons, π-mesons . . . . . .).

Fundamental fields:
Quarks: Spin $\frac{1}{2}$ fields with colour quantum numbers, interacting through Gluons.
Theory invariant under $SU(3)$ gauge transformations.

Cf. with Quantum Electrodynamics: electrons and photons, gauge group $U(1)$.

Confinement:To the quarks fields correspond no states in the Fock space of hadrons.

Problem of all realistic Quantum Field Theories:
The only analytically tractable treatment is perturbation theory around the free fields.

Not adequate for the effects of strong interactions, like confining the quarks.

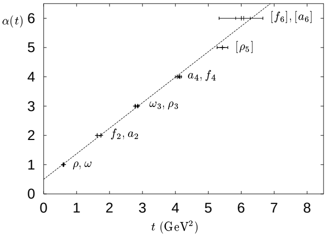
But even in QED: Need for a semiclassical treatment of bound state problems (Schrödinger, Dirac equation)

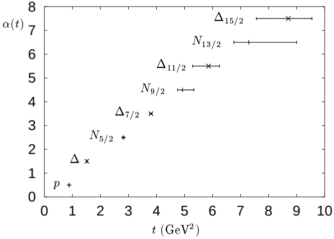
Formidable task: Find semiclassical equations for QCD!

Not completely unrealistic:

• **Qualitative success of non-relativistic quark model**
(based on Schroedinger equation)

• **Striking regularities in the hadronic spectra**
Notably Regge Trajectories



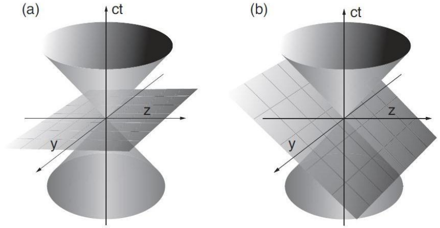


From Donnachie et al. “QCD and Pomeron Physics”

• **Light Front Quantization**

adequate framework in relativistic theories

Dirac, Rev. Mod. Phys. 21, 392 (1949), Brodsky *et al.* ...



Instant-form: $x^0 = 0$ Light Front-form: $x^0 + x^3 = 0$

For the Fock state of two massless quarks:

$$\left(-\partial_\zeta^2 + \frac{4L^2-1}{4\zeta^2} + U(\zeta)\right)\psi(\zeta) = P^2\psi(\zeta)$$

ζ: essentially the separation of the quarks in the transverse (1-2) plane, L: longitud. angular momentum

$U(\zeta)$ comprises all interactions, including those with higher Fock states

Eigenvalues:Squared hadron masses

Strong boost: Maldacena Conjecture Adv. Theor. Math. Phys. **2**, 231 (1998):

Gauge-QFT $d = 4 \sim$ Classical gravitational theory $d = 5$

Generating functional of the gauge QFT is given by the minimum of the classical action of the gravitational theory at the 4-dim border of the 5-dim space

Graviation: AdS_5 metric , $ds^2 = \frac{R^2}{z^2}\left(\sum_{i=0}^{3} dx_i\, dx^i - dz^2\right)$

Poincaré coordinates. $z = x^5$ holographic coordinate,
$z = 0$ border to 4-dim. space $R(1,3)$

QFT: greatly oversymmetrized:
conformal supersymmetric gauge theory

bottom-up approach: Start from realistic 4-dim Theory.

Here: Light-Front Holography

G. F. de Téramond and S. J. Brodsky Phys. Rev. Lett. **102**, 081601 (2009)

Consider scalar field in AdS_5:

Action:

$$S = \int d^4x\, dz\, \sqrt{|g|}\, \left(g^{MN}\partial_M\Phi(x,z)\partial_N\Phi(x,z) - \mu^2\Phi(x,z)^2\right)$$

With $\Phi(x,z) = e^{iPx}\Phi(z)$ e.o.m. can be brought into the form:

$$\left(-\partial_z^2 + \frac{4(\mu R)^2+16-1}{4z^2}\right)\phi(z) = P^2\phi(z)$$

Compare with LF equation:

$$\left(-\partial_\zeta^2 + \frac{4L^2-1}{4\zeta^2} + U(\zeta)\right)\psi(\zeta) = P^2\psi(\zeta)$$

same structure: $z \to \zeta \quad (\mu R)^2 + 4 \to L^2 \quad$ but $U(\zeta) = 0$

No wonder: AdS_5 maximally symmetric,
15 isometries $\to$ Conf($R^{1,3}$),(10 Poincaré + 4 inversions + dilatation)

Conformal symmetry: No scale $\to$ no dicrete spectrum

Way out: Distort maximal symmetry!

$$S = \int d^4x\, dz\, \sqrt{|g|}\, \left(g^{MN}\partial_M\Phi(x,z)\partial_N\Phi(x,z) - \mu^2\Phi(x,z)^2\right)$$

$$\Downarrow$$

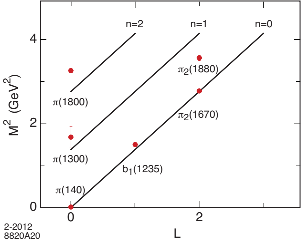
$$S = \int d^4x\, dz\, \sqrt{|g|}e^{\varphi(z)}\, \left(g^{MN}\partial_M\Phi(x,z)\partial_N\Phi(x,z) - \mu^2\Phi(x,z)^2\right)$$

$$U(z) = \frac{1}{4}(\varphi'(z))^2 - \frac{3}{z}\varphi'(z) + \frac{1}{2}\varphi''(z)$$

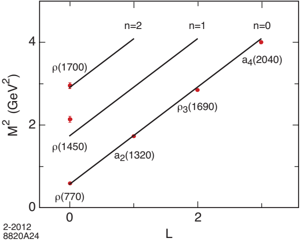
Soft wall model

Karch, Katz, Son and Stephanov, Phys. Rev. D **74**, 015005 (2006),
G. F. de Téramond and S. J. Brodsky, Nucl. Phys. Proc. Suppl. **199**, 89 (2010) ; arXiv:1203.4025, . . .
G. F. de Téramond, H. G. D. and S. J. Brodsky, Phys. Rev. D **87**, 075005 (2013)

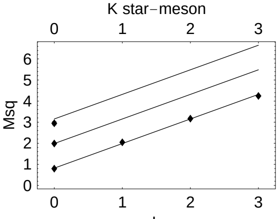
$$\varphi(z) = \lambda\, z^2,\ \Rightarrow\ U(\zeta) = \lambda^2\zeta^2 - 2\lambda$$



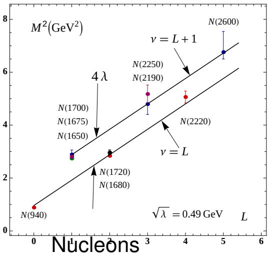
$S = 0$ mesons



$J = L + 1$ mesons



K^* mesons



Nucleons

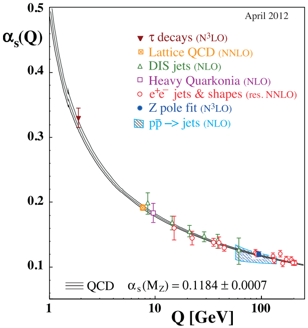
from de Téramond and Brodsky, ArXiv:1203.4025; Brodsky, de Téramond, HGD, Josh Erlich, Phys. Rep. to appear.

Choice of $\varphi(z) = \lambda z^2$ only motivated by phenomenology. Some deeper reason?

Conformal symmetry.

Classical QCD-Lagrangian with massless quarks conformally invariant.

Need for renormalization $\Rightarrow$ introduces a scale Λ_{QCD}



Scale dependence of QCD coupling in perturbative regime (small distance = large momentum scales).

$$Q^2 \frac{d\alpha_s(Q^2)}{dQ^2} = -\sum_{i=0} \frac{\beta_i}{4\pi} \alpha_s^{2+i} + \dots$$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0} \frac{1}{\log(Q^2/\Lambda_{\rm QCD})} + \dots$$

from PDG

Indications: At large distances, small Q, α_s becomes constant again. Restauration of conformal symmetry?

Fresh look at conformal symmetry

Our goal: Find a semiclassical approximarion, i.e. approximate QFT by Quantum Mechanics

Conformal QM = conformal QFT in 1 dimension.

V. de Alfaro, S. Fubini and G. Furlan Nuovo Cim. A **34**, 569 (1976)

$$S_{conf} = \frac{1}{2}\int dt \left(\dot{Q}(t)^2 - \frac{g}{Q(2)^2}\right) \; H = \frac{1}{2}\left(\dot{Q}^2 + \frac{g}{Q^2}\right)$$

$$P = \frac{\delta S}{\delta \dot{Q}} = \dot{Q} \to \; [Q, \dot{Q}] = i \overset{\text{Schrödinger}}{\to} \; Q(0) = r, \; \dot{Q}(0) = -i\partial_r$$

$$H\psi(r) = \frac{1}{2}\left(-\partial_r^2 + \frac{g}{r^2}\right)\psi(r)$$

again back to free equation.

$$\text{AdS: } \left(-\partial_z^2 + \frac{4(\mu R)^2 + 16 - 1}{4z^2}\right)\phi(z) = P^2\phi(z)$$

$$\text{LF : } Big(-\partial_\zeta^2 + \frac{4L^2-1}{4\zeta^2} + \underbrace{U(\zeta)}_{=0}\Big)\psi(\zeta) = P^2\psi(\zeta)$$

But, as stressed by de Alfaro, Fubini and Furlan:

There are besides H, the generator of translations t two more constants of motion, namely the two other Noether currents of the conformal action S_{conf}

D for dilatations, $t \to t(1+\epsilon)$ and
K for special conformal transformation $t \to \frac{t}{1-\epsilon t}$

Allows to construct a generalized Hamiltonian:

$G = H + w\,K + v\,D$ translation in τ with $d\tau = \frac{dt}{1+vt+wt^2}$

Schrödinger

$$G\,\psi(r) = \frac{1}{2}\left(-\partial_r^2 + \frac{g}{r^2} + w\,r^2 + \frac{i\,v}{2}(r\partial_r + \partial_r r)\right)\psi(r)$$

distorted Ads:

$$\left(-\partial_z^2 + \frac{4(\mu R)^2+16-1}{4z^2} + \frac{1}{4}(\varphi'(z))^2 - \frac{3}{z}\varphi'(z) + \frac{1}{2}\varphi''(z)\right)\phi(z) = P^2\phi(z)$$

LF : $\left(-\partial_\zeta^2 + \frac{4L^2-1}{4\zeta^2} + U(\zeta)\right)\psi(\zeta) = P^2\psi(\zeta)$

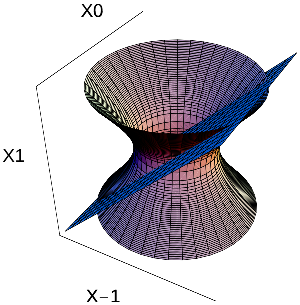
The conformal Lagrangian S_{conf} implies unambiguously :

$$\varphi(z) = \lambda\,z^2 \qquad U(\zeta) = \lambda\,\zeta^2$$

Back to geometry;

The conformal group $Conf(R^1)$ is isomorphic to the Lorentz group $S0(2,1)$ and therefore to the isometries of AdS_2.

Best seen by embedding AdS_2 into a three-dimensional space:



$$X_{-1}^2 + X_0^2 - X_1^2 = R^2$$

Poincaré coordinates:

$$z = \frac{R^2}{X_{-1} - X_1} \qquad x^0 = \frac{X_0(X_{-1} - X_1)}{R}$$

Generators of $S0(2,1)$

Rotation J^{-10} , boosts J^{01} $J^{-1,1}$

are the isometries of AdS_2

Isomorphism

$SO(2,1) \sim Conf(R^1)$:

$$a\,H = J^{-10} - J^{01},$$
$$\tfrac{1}{a}K = J^{-10} + J^{01},$$
$$D = J^{-1,1}$$

The confining G

$G = H + wK$:

$$\tfrac{1+\theta}{2}\, a\; G = J^{-10} - \theta\, J^{01}$$

Schrödinger: $G =$

$$\tfrac{1}{2}\Big(-\partial_r^2 + \tfrac{g}{r^2} + \tfrac{1}{a^2}\tfrac{1-\theta}{1+\theta} r^2 \Big)$$